Answers to Odd-Numbered Exercises

Chapter 1

Section 1.1, page 10

1. The solution is $(x_1, x_2) = (-8, 3)$, or simply $(-8, 3)$.

3. $(4/7, 9/7)$

5. Replace row 2 by its sum with 3 times row 3, and then replace row 1 by its sum with -5 times row 3.

7. The solution set is empty.

9. $(4, 8, 5, 2)$ **11.** Inconsistent

13. $(5, 3, -1)$ **15.** Consistent

17. The three lines have one point in common.

19. $h \neq 2$ **21.** All h

23. Mark a statement True only if the statement is *always* true. Giving you the answers here would defeat the purpose of the true–false questions, which is to help you learn to read the text carefully. The *Study Guide* will tell you where to look for the answers, but you should not consult it until you have made an honest attempt to find the answers yourself.

25. $k + 2g + h = 0$

27. The row reduction of $\begin{bmatrix} 1 & 3 & f \\ c & d & g \end{bmatrix}$ to $\begin{bmatrix} 1 & 3 & f \\ 0 & d-3c & g-cf \end{bmatrix}$ shows that $d - 3c$ must be nonzero, since f and g are arbitrary. Otherwise, for some choices of f and g the second row could correspond to an equation of the form $0 = b$, where b is nonzero. Thus $d \neq 3c$.

29. Swap row 1 and row 2; swap row 1 and row 2.

31. Replace row 3 by row 3 + (-4) row 1; replace row 3 by row 3 + (4) row 1.

33.
$$\begin{aligned} 4T_1 - T_2 - T_4 &= 30 \\ -T_1 + 4T_2 - T_3 &= 60 \\ -T_2 + 4T_3 - T_4 &= 70 \\ -T_1 - T_3 + 4T_4 &= 40 \end{aligned}$$

Section 1.2, page 21

1. Reduced echelon form: a and b. Echelon form: d. Not echelon: c.

3. $\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Pivot cols 1 and 2: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$.

5. $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix}$, $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

7. $\begin{cases} x_1 = -5 - 3x_2 \\ x_2 \text{ is free} \\ x_3 = 3 \end{cases}$ **9.** $\begin{cases} x_1 = 4 + 5x_3 \\ x_2 = 5 + 6x_3 \\ x_3 \text{ is free} \end{cases}$

11. $\begin{cases} x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$

13. $\begin{cases} x_1 = 5 + 3x_5 \\ x_2 = 1 + 4x_5 \\ x_3 \text{ is free} \\ x_4 = 4 - 9x_5 \\ x_5 \text{ is free} \end{cases}$

Note: The *Study Guide* discusses the common mistake $x_3 = 0$.

15. **a.** Consistent, with a unique solution

b. Inconsistent

17. $h = 7/2$

19. **a.** Inconsistent when $h = 2$ and $k \neq 8$

b. A unique solution when $h \neq 2$

c. Many solutions when $h = 2$ and $k = 8$

21. Read the text carefully, and write your answers before you consult the *Study Guide*. Remember, a statement is true only if it is true in all cases.

23. Yes. The system is consistent because with three pivots, there must be a pivot in the third (bottom) row of the coefficient matrix. The reduced echelon form cannot contain a row of the form $[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$.

25. If the coefficient matrix has a pivot position in every row, then there is a pivot position in the bottom row, and there is no room for a pivot in the augmented column. So, the system is consistent, by Theorem 2.

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27. If a linear system is consistent, then the solution is unique if and only if *every column in the coefficient matrix is a pivot column; otherwise, there are infinitely many solutions*.

29. An underdetermined system always has more variables than equations. There cannot be more basic variables than there are equations, so there must be at least one free variable. Such a variable may be assigned infinitely many different values. If the system is consistent, each different value of a free variable will produce a different solution.

31. Yes, a system of linear equations with more equations than unknowns can be consistent. The following system has a solution ($x_1 = x_2 = 1$):

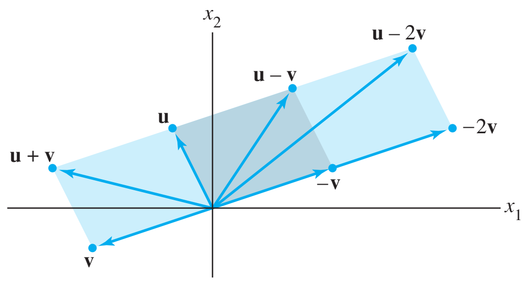
$$\begin{aligned} x_1 + \ \ x_2 &= 2 \\ x_1 - \ \ x_2 &= 0 \\ 3x_1 + 2x_2 &= 5 \end{aligned}$$

33. [**M**] $p(t) = 7 + 6t - t^2$

Section 1.3, page 32

1. $\begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

3.



5. $x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$,

$$\begin{bmatrix} 6x_1 \\ -x_1 \\ 5x_1 \end{bmatrix} + \begin{bmatrix} -3x_2 \\ 4x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}, \begin{bmatrix} 6x_1 - 3x_2 \\ -x_1 + 4x_2 \\ 5x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$$

$$\begin{aligned} 6x_1 - 3x_2 &= 1 \\ -x_1 + 4x_2 &= -7 \\ 5x_1 \qquad &= -5 \end{aligned}$$

Usually the intermediate steps are not displayed.

7. $\mathbf{a} = \mathbf{u} - 2\mathbf{v}$, $\mathbf{b} = 2\mathbf{u} - 2\mathbf{v}$, $\mathbf{c} = 2\mathbf{u} - 3.5\mathbf{v}$, $\mathbf{d} = 3\mathbf{u} - 4\mathbf{v}$

9. $x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

11. Yes, $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

13. No, $\mathbf{b}$ is *not* a linear combination of the columns of A.

15. Noninteger weights are acceptable, of course, but some simple choices are $0 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 = \mathbf{0}$, and

$$1 \cdot \mathbf{v}_1 + 0 \cdot \mathbf{v}_2 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix}, 0 \cdot \mathbf{v}_1 + 1 \cdot \mathbf{v}_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

$$1 \cdot \mathbf{v}_1 + 1 \cdot \mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}, 1 \cdot \mathbf{v}_1 - 1 \cdot \mathbf{v}_2 = \begin{bmatrix} 12 \\ -2 \\ -6 \end{bmatrix}$$

17. $h = -17$

19. Span $\{\mathbf{v}_1, \mathbf{v}_2\}$ is the set of points on the line through $\mathbf{v}_1$ and $\mathbf{0}$.

21. *Hint:* Show that $\begin{bmatrix} 2 & 2 & h \\ -1 & 1 & k \end{bmatrix}$ is consistent for all h and k. Explain what this calculation shows about Span $\{\mathbf{u}, \mathbf{v}\}$.

23. Before you consult your *Study Guide*, read the entire section carefully. Pay special attention to definitions and theorem statements, and note any remarks that precede or follow them.

25. **a.** No, three **b.** Yes, infinitely many

c. $\mathbf{a}_1 = 1 \cdot \mathbf{a}_1 + 0 \cdot \mathbf{a}_2 + 0 \cdot \mathbf{a}_3$

27. **a.** $5\mathbf{v}_1$ is the output of 5 day's operation of mine #1.

b. The total output is $x_1\mathbf{v}_1 + x_2\mathbf{v}_2$, so x_1 and x_2 should satisfy $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 = \begin{bmatrix} 150 \\ 2825 \end{bmatrix}$.

c. [**M**] 1.5 days for mine #1 and 4 days for mine #2

29. (1.3, .9, 0)

31. **a.** $\begin{bmatrix} 10/3 \\ 2 \end{bmatrix}$

b. Add 3.5 g at (0, 1), add .5 g at (8, 1), and add 2 g at (2, 4).

33. Review Practice Problem 1 and then *write* a solution. The *Study Guide* has a solution.

Section 1.4, page 40

1. The product is not defined because the number of columns (2) in the 3×2 matrix does not match the number of entries (3) in the vector.

3. $A\mathbf{x} = \begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 2 \cdot \begin{bmatrix} 6 \\ -4 \\ 7 \end{bmatrix} - 3 \cdot \begin{bmatrix} 5 \\ -3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 12 \\ -8 \\ 14 \end{bmatrix} + \begin{bmatrix} -15 \\ 9 \\ -18 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$, and

$A\mathbf{x} = \begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 6 \cdot 2 + 5 \cdot (-3) \\ (-4) \cdot 2 + (-3) \cdot (-3) \\ 7 \cdot 2 + 6 \cdot (-3) \end{bmatrix}$

$= \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$. Show your work here and for Exercises 4–6, but thereafter perform the calculations mentally.

5. $5 \cdot \begin{bmatrix} 5 \\ -2 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 \\ -7 \end{bmatrix} + 3 \cdot \begin{bmatrix} -8 \\ 3 \end{bmatrix} - 2 \cdot \begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$

7. $\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

9. $x_1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$ and

$\begin{bmatrix} 3 & 1 & -5 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

13. Yes. (Justify your answer.)

u are here

15. The equation $A\mathbf{x} = \mathbf{b}$ is not consistent when $3b_1 + b_2$ is nonzero. (Show your work.) The set of **b** for which the equation *is* consistent is a line through the origin—the set of all points (b_1, b_2) satisfying $b_2 = -3b_1$.

17. Only three rows contain a pivot position. The equation $A\mathbf{x} = \mathbf{b}$ does *not* have a solution for each **b** in $\mathbb{R}^4$, by Theorem 4.

19. The work in Exercise 17 shows that statement (d) in Theorem 4 is false. So all four statements in Theorem 4 are false. Thus, not all vectors in $\mathbb{R}^4$ can be written as a linear combination of the columns of A. Also, the columns of A do *not* span $\mathbb{R}^4$.

21. The matrix $[\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]$ does not have a pivot in each row, so the columns of the matrix do not span $\mathbb{R}^4$, by Theorem 4. That is, $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ does not span $\mathbb{R}^4$.

23. Read the text carefully and try to mark each exercise statement True or False before you consult the *Study Guide*. Several parts of Exercises 23 and 24 are *implications* of the form

"If ⟨statement 1⟩, then ⟨statement 2⟩"

or equivalently,

"⟨statement 2⟩, if ⟨statement 1⟩"

Mark such an implication as True if ⟨statement 2⟩ is true in all cases when ⟨statement 1⟩ is true.

25. $c_1 = -3, c_2 = -1, c_3 = 2$

27. $Q\mathbf{x} = \mathbf{v}$, where $Q = [\mathbf{q}_1 \quad \mathbf{q}_2 \quad \mathbf{q}_3]$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Note: If your answer is the equation $A\mathbf{x} = \mathbf{b}$, you must specify what A and **b** are.

29. *Hint:* Start with any 3×3 matrix B in echelon form that has three pivot positions.

31. *Write* your solution before you check the *Study Guide*.

33. *Hint:* How many pivot columns does A have? Why?

35. Given $A\mathbf{x}_1 = \mathbf{y}_1$ and $A\mathbf{x}_2 = \mathbf{y}_2$, you are asked to show that the equation $A\mathbf{x} = \mathbf{w}$ has a solution, where $\mathbf{w} = \mathbf{y}_1 + \mathbf{y}_2$. Observe that $\mathbf{w} = A\mathbf{x}_1 + A\mathbf{x}_2$ and use Theorem 5(a) with $\mathbf{x}_1$ and $\mathbf{x}_2$ in place of **u** and **v**, respectively. That is, $\mathbf{w} = A\mathbf{x}_1 + A\mathbf{x}_2 = A(\mathbf{x}_1 + \mathbf{x}_2)$. So the vector $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2$ is a solution of $\mathbf{w} = A\mathbf{x}$.

37. [**M**] The columns do not span $\mathbb{R}^4$.

39. [**M**] The columns span $\mathbb{R}^4$.

41. [**M**] Delete column 4 of the matrix in Exercise 39. It is also possible to delete column 3 instead of column 4.

Section 1.5, page 48

1. The system has a nontrivial solution because there is a free variable, x_3.

3. The system has a nontrivial solution because there is a free variable, x_3.

5. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

7. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}$

9. $\mathbf{x} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

11. *Hint:* The system derived from the *reduced* echelon form is

$$\begin{aligned} x_1 - 4x_2 \qquad\qquad + 5x_6 &= 0 \\ x_3 \qquad - x_6 &= 0 \\ x_5 - 4x_6 &= 0 \\ 0 &= 0 \end{aligned}$$

The basic variables are x_1, x_3, and x_5. The remaining variables are free. The *Study Guide* discusses two mistakes that are often made on this type of problem.

13. $\mathbf{x} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix} = \mathbf{p} + x_3\mathbf{q}$. Geometrically, the solution set is the line through $\begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$ parallel to $\begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix}$.

15. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$. The solution set is the line through $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, parallel to the line that is the solution set of the homogeneous system in Exercise 5.

17. Let $\mathbf{u} = \begin{bmatrix} -9 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$. The solution of the homogeneous equation is $\mathbf{x} = x_2\mathbf{u} + x_3\mathbf{v}$, the plane through the origin spanned by $\mathbf{u}$ and $\mathbf{v}$. The solution set of the nonhomogeneous system is $\mathbf{x} = \mathbf{p} + x_2\mathbf{u} + x_3\mathbf{v}$, the plane through $\mathbf{p}$ parallel to the solution set of the homogeneous equation.

19. $\mathbf{x} = \mathbf{a} + t\mathbf{b}$, where t represents a parameter, or
$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 3 \end{bmatrix}$, or $\begin{cases} x_1 = -2 - 5t \\ x_2 = 3t \end{cases}$

21. $\mathbf{x} = \mathbf{p} + t(\mathbf{q} - \mathbf{p}) = \begin{bmatrix} 2 \\ -5 \end{bmatrix} + t \begin{bmatrix} -5 \\ 6 \end{bmatrix}$

23. It is important to read the text carefully and write your answers. After that, check the *Study Guide*, if necessary.

25. $A\mathbf{v}_h = A(\mathbf{w} - \mathbf{p}) = A\mathbf{w} - A\mathbf{p} = \mathbf{b} - \mathbf{b} = \mathbf{0}$

27. When A is the 3×3 zero matrix, *every* $\mathbf{x}$ in $\mathbb{R}^3$ satisfies $A\mathbf{x} = \mathbf{0}$. So the solution set is all vectors in $\mathbb{R}^3$.

29. a. When A is a 3×3 matrix with three pivot positions, the equation $A\mathbf{x} = \mathbf{0}$ has no free variables and hence has no nontrivial solution.

b. With three pivot positions, A has a pivot position in each of its three rows. By Theorem 4 in Section 1.4, the equation $A\mathbf{x} = \mathbf{b}$ has a solution for every possible $\mathbf{b}$. The word "possible" in the exercise means that the only vectors considered in this case are those in $\mathbb{R}^3$, because A has three rows.

31. a. When A is a 3×2 matrix with two pivot positions, each column is a pivot column. So the equation $A\mathbf{x} = \mathbf{0}$ has no free variables and hence no nontrivial solution.

b. With two pivot positions and three rows, A cannot have a pivot in every row. So the equation $A\mathbf{x} = \mathbf{b}$ cannot have a solution for every possible $\mathbf{b}$ (in $\mathbb{R}^3$), by Theorem 4 in Section 1.4.

33. One answer: $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

35. Your example should have the property that the sum of the entries in each row is zero. Why?

37. One answer is $A = \begin{bmatrix} 1 & -4 \\ 1 & -4 \end{bmatrix}$. The *Study Guide* shows how to analyze the problem in order to construct A. If $\mathbf{b}$ is any vector *not* a multiple of the first column of A, then the solution set of $A\mathbf{x} = \mathbf{b}$ is empty and thus cannot be formed by translating the solution set of $A\mathbf{x} = \mathbf{b}$. This does not contradict Theorem 6, because that theorem applies when the equation $A\mathbf{x} = \mathbf{b}$ has a nonempty solution set.

39. If c is a scalar, then $A(c\mathbf{u}) = cA\mathbf{u}$, by Theorem 5(b) in Section 1.4. If $\mathbf{u}$ satisfies $A\mathbf{x} = \mathbf{0}$, then $A\mathbf{u} = \mathbf{0}$, $cA\mathbf{u} = c \cdot \mathbf{0} = \mathbf{0}$, and so $A(c\mathbf{u}) = \mathbf{0}$.

Section 1.6, page 55

1. The general solution is $p_{\text{Goods}} = .875p_{\text{Services}}$, with p_{Services} free. One equilibrium solution is $p_{\text{Services}} = 1000$ and $p_{\text{Goods}} = 875$. Using fractions, the general solution could be written $p_{\text{Goods}} = (7/8)p_{\text{Services}}$, and a natural choice of prices might be $p_{\text{Services}} = 80$ and $p_{\text{Goods}} = 70$. Only the *ratio* of the prices is important. The economic equilibrium is unaffected by a proportional change in prices.

3. a.

	Distribution of Output From:				
	C&M	F&P	Mach.		
Output	↓	↓	↓	Input	Purchased By:
	.2	.8	.4	→	C&M
	.3	.1	.4	→	F&P
	.5	.1	.2	→	Mach.

b. $\begin{bmatrix} .8 & -.8 & -.4 & 0 \\ -.3 & .9 & -.4 & 0 \\ -.5 & -.1 & .8 & 0 \end{bmatrix}$

c. **[M]** $p_{\text{Chemicals}} = 141.7$, $p_{\text{Fuels}} = 91.7$, $p_{\text{Machinery}} = 100$. To two significant figures, $p_{\text{Chemicals}} = 140$, $p_{\text{Fuels}} = 92$, $p_{\text{Machinery}} = 100$.

5. $B_2S_3 + 6H_2O \to 2H_3BO_3 + 3H_2S$

7. $3NaHCO_3 + H_3C_6H_5O_7 \to Na_3C_6H_5O_7 + 3H_2O + 3CO_2$

9. **[M]** $15PbN_6 + 44CrMn_2O_8 \to$
$5Pb_3O_4 + 22Cr_2O_3 + 88MnO_2 + 90NO$

11. $\begin{cases} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$ The largest value of x_3 is 20.

13. a. $\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 \text{ is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 \text{ is free} \end{cases}$ **b.** $\begin{cases} x_2 = 50 \\ x_3 = 40 \\ x_4 = 50 \\ x_5 = 60 \end{cases}$

Section 1.7, page 61

Justify your answers to Exercises 1–22.

1. Lin. indep. **3.** Lin. depen.

5. Lin. indep. **7.** Lin. depen.

9. a. No h **b.** All h

11. $h = 6$ **13.** All h

15. Lin. depen. **17.** Lin. depen. **19.** Lin. indep.

21. If you consult your *Study Guide* before you make a good effort to answer the true-false questions, you will destroy most of their value.

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23. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \end{bmatrix}$ **25.** $\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

27. All five columns of the 7×5 matrix A must be pivot columns. Otherwise, the equation $A\mathbf{x} = \mathbf{0}$ would have a free variable, in which case the columns of A would be linearly dependent.

29. A: Any 3×2 matrix with two nonzero columns such that neither column is a multiple of the other. In this case, the columns are linearly independent, and so the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
B: Any 3×2 matrix with one column a multiple of the other.

31. $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

33. True, by Theorem 7. (The *Study Guide* adds another justification.)

35. False. The vector $\mathbf{v}_1$ could be the zero vector.

37. True. A linear dependence relation among $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ may be extended to a linear dependence relation among $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ by placing a zero weight on $\mathbf{v}_4$.

39. You should be able to work this important problem without help. *Write* your solution before you consult the *Study Guide*.

41. [**M**] $B = \begin{bmatrix} 8 & -3 & 2 \\ -9 & 4 & -7 \\ 6 & -2 & 4 \\ 5 & -1 & 10 \end{bmatrix}$. Other choices are possible.

43. [**M**] Each column of A that is not a column of B is in the set spanned by the columns of B.

Section 1.8, page 69

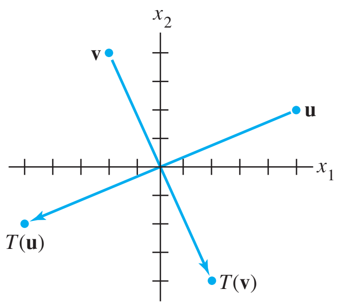
1. $\begin{bmatrix} 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 2a \\ 2b \end{bmatrix}$ **3.** $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, unique solution

5. $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, not unique **7.** $a = 5, b = 6$

9. $\mathbf{x} = x_3 \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

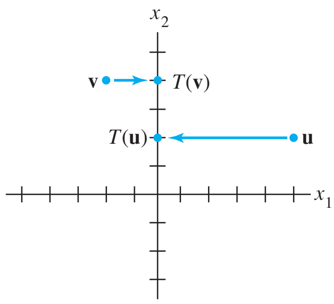
11. Yes, because the system represented by $[\,A \quad \mathbf{b}\,]$ is consistent.

13.



A reflection through the origin

15.



A projection onto the x_2-axis.

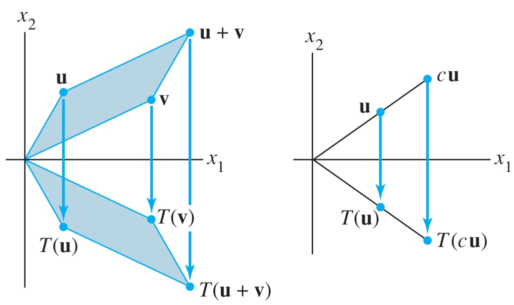
17. $\begin{bmatrix} 6 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix}$ **19.** $\begin{bmatrix} 13 \\ 7 \end{bmatrix}, \begin{bmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$

21. Read the text carefully and write your answers before you check the *Study Guide*. Notice that Exercise 21(e) is a sentence of the form

"⟨statement 1⟩ if and only if ⟨statement 2⟩"

Mark such a sentence as True if ⟨statement 1⟩ is true whenever ⟨statement 2⟩ is true *and* also ⟨statement 2⟩ is true whenever ⟨statement 1⟩ is true.

23.



25. *Hint:* Show that the image of a line (that is, the set of images of all points on a line) can be represented by the parametric equation of a line.

27. **a.** The line through $\mathbf{p}$ and $\mathbf{q}$ is parallel to $\mathbf{q} - \mathbf{p}$. (See Exercises 21 and 22 in Section 1.5.) Since $\mathbf{p}$ is on the line, the equation of the line is $\mathbf{x} = \mathbf{p} + t(\mathbf{q} - \mathbf{p})$. Rewrite this as $\mathbf{x} = \mathbf{p} - t\mathbf{p} + t\mathbf{q}$ and $\mathbf{x} = (1 - t)\mathbf{p} + t\mathbf{q}$.

b. Consider $\mathbf{x} = (1 - t)\mathbf{p} + t\mathbf{q}$ for t such that $0 \le t \le 1$. Then, by linearity of T, for $0 \le t \le 1$

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EQA

$$T(\mathbf{x}) = T((1-t)\mathbf{p} + t\mathbf{q}) = (1-t)T(\mathbf{p}) + tT(\mathbf{q}) \quad (*)$$

If $T(\mathbf{p})$ and $T(\mathbf{q})$ are distinct, then (*) is the equation for the line segment between $T(\mathbf{p})$ and $T(\mathbf{q})$, as shown in part (a). Otherwise, the set of images is just the single point $T(\mathbf{p})$, because

$$(1-t)T(\mathbf{p}) + tT(\mathbf{q}) = (1-t)T(\mathbf{p}) + tT(\mathbf{p}) = T(\mathbf{p})$$

29. a. When $b = 0$, $f(x) = mx$. In this case, for all x, y in $\mathbb{R}$ and all scalars c and d,

$$\begin{aligned} f(cx+dy) &= m(cx+dy) = mcx + mdy \\ &= c(mx) + d(my) = c \cdot f(x) + d \cdot f(y) \end{aligned}$$

This shows that f is linear.

b. When $f(x) = mx + b$, with b nonzero, $f(0) = m(0) + b = b \neq 0$.

c. In calculus, f is called a "linear function" because the graph of f is a line.

31. *Hint:* Since $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, you can write a certain equation and work with it.

33. One possibility is to show that T does not map the zero vector into the zero vector, something that every linear transformation *does* do: $T(0,0) = (0,4,0)$.

35. Take $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ and let c and d be scalars. Then

$$c\mathbf{u} + d\mathbf{v} = (cu_1 + dv_1, cu_2 + dv_2, cu_3 + dv_3)$$

The transformation T is linear because

$$\begin{aligned} T(c\mathbf{u} + d\mathbf{v}) &= (cu_1 + dv_1, cu_2 + dv_2, -(cu_3 + dv_3)) \\ &= (cu_1 + dv_1, cu_2 + dv_2, -cu_3 - dv_3) \\ &= (cu_1, cu_2, -cu_3) + (dv_1, dv_2, -dv_3) \\ &= c(u_1, u_2, -u_3) + d(v_1, v_2, -v_3) \\ &= cT(\mathbf{u}) + dT(\mathbf{v}) \end{aligned}$$

37. **[M]** All multiples of $(7, 9, 0, 2)$

39. **[M]** Yes. One choice for $\mathbf{x}$ is $(4, 7, 1, 0)$.

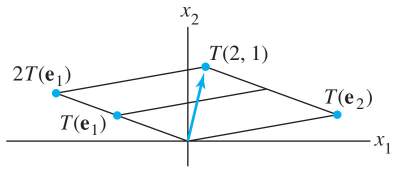
Section 1.9, page 79

1. $\begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix}$ **3.** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ **5.** $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

7. $\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$ **9.** $\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

11. The described transformation T maps $\mathbf{e}_1$ into $-\mathbf{e}_1$ and maps $\mathbf{e}_2$ into $-\mathbf{e}_2$. A rotation through π radians also maps $\mathbf{e}_1$ into $-\mathbf{e}_1$ and maps $\mathbf{e}_2$ into $-\mathbf{e}_2$. Since a linear transformation is completely determined by what it does to the columns of the identity matrix, the rotation transformation has the same effect as T on every vector in $\mathbb{R}^2$.

13.



15. $\begin{bmatrix} 3 & 0 & -2 \\ 4 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ **17.** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$ **21.** $\mathbf{x} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$

23. Answer the questions before checking the *Study Guide*.

Justify your answers to Exercises 25–28.

25. Not one-to-one and does not map $\mathbb{R}^4$ onto $\mathbb{R}^4$

27. Not one-to-one but maps $\mathbb{R}^3$ onto $\mathbb{R}^2$

29. $\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{bmatrix}$

31. n. (Explain why, and then check the *Study Guide*).

33. *Hint:* If $\mathbf{e}_j$ is the jth column of I_n, then $B\mathbf{e}_j$ is the jth column of B.

35. *Hint:* Is it possible that $m > n$? What about $m < n$?

37. **[M]** No. (Explain why.)

39. **[M]** No. (Explain why.)

Section 1.10, page 87

1. a. $x_1 \begin{bmatrix} 110 \\ 4 \\ 20 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 130 \\ 3 \\ 18 \\ 5 \end{bmatrix} = \begin{bmatrix} 295 \\ 9 \\ 48 \\ 8 \end{bmatrix}$, where x_1 is the number of servings of Cheerios and x_2 is the number of servings of 100% Natural Cereal.

b. $\begin{bmatrix} 110 & 130 \\ 4 & 3 \\ 20 & 18 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 295 \\ 9 \\ 48 \\ 8 \end{bmatrix}$. Mix 1.5 servings of Cheerios together with 1 serving of 100% Natural Cereal.

3. a. She should mix .99 serving of Mac and Cheese, 1.54 servings of broccoli, and .79 serving of chicken to get her desired nutritional content.

b. She should mix 1.09 servings of shells and white cheddar, .88 serving of broccoli, and 1.03 servings of chicken to get her desired nutritional content. Notice that this mix contains significantly less broccoli, so she should like it better.

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5. $R\mathbf{i} = \mathbf{v}$, $\begin{bmatrix} 11 & -5 & 0 & 0 \\ -5 & 10 & -1 & 0 \\ 0 & -1 & 9 & -2 \\ 0 & 0 & -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 50 \\ -40 \\ 30 \\ -30 \end{bmatrix}$

[**M**]: $\mathbf{i} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 3.68 \\ -1.90 \\ 2.57 \\ -2.49 \end{bmatrix}$

7. $R\mathbf{i} = \mathbf{v}$, $\begin{bmatrix} 12 & -7 & 0 & -4 \\ -7 & 15 & -6 & 0 \\ 0 & -6 & 14 & -5 \\ -4 & 0 & -5 & 13 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 40 \\ 30 \\ 20 \\ -10 \end{bmatrix}$

[**M**]: $\mathbf{i} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 11.43 \\ 10.55 \\ 8.04 \\ 5.84 \end{bmatrix}$

9. $\mathbf{x}_{k+1} = M\mathbf{x}_k$ for $k = 0, 1, 2, \ldots,$ where

$M = \begin{bmatrix} .93 & .05 \\ .07 & .95 \end{bmatrix}$ and $\mathbf{x}_0 = \begin{bmatrix} 800{,}000 \\ 500{,}000 \end{bmatrix}$.

The population in 2017 (for $k = 2$) is $\mathbf{x}_2 = \begin{bmatrix} 741{,}720 \\ 558{,}280 \end{bmatrix}$.

11. a. $M = \begin{bmatrix} .98033 & .00179 \\ .01967 & .99821 \end{bmatrix}$

b. [**M**] $\mathbf{x}_{10} = \begin{bmatrix} 35.729 \\ 278.18 \end{bmatrix}$

13. [**M**]

a. The population of the city decreases. After 7 years, the populations are about equal, but the city population continues to decline. After 20 years, there are only 417,000 persons in the city (417,456 rounded off). However, the changes in population seem to grow smaller each year.

b. The city population is increasing slowly, and the suburban population is decreasing. After 20 years, the city population has grown from 350,000 to about 370,000.

Chapter 1 Supplementary Exercises, page 89

1.

a. F	**b.** F	**c.** T	**d.** F	**e.** T	
f. T	**g.** F	**h.** F	**i.** T	**j.** F	
k. T	**l.** F	**m.** T	**n.** T	**o.** T	
p. T	**q.** F	**r.** T	**s.** F	**t.** T	
u. F	**v.** F	**w.** T	**x.** T	**y.** T	**z.** F

3. a. Any consistent linear system whose echelon form is

$$\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b. Any consistent linear system whose reduced echelon form is I_3.

c. Any inconsistent linear system of three equations in three variables.

5. a. The solution set: (i) is empty if $h = 12$ and $k \neq 2$; (ii) contains a unique solution if $h \neq 12$; (iii) contains infinitely many solutions if $h = 12$ and $k = 2$.

b. The solution set is empty if $k + 3h = 0$; otherwise, the solution set contains a unique solution.

7. a. Set $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ -5 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. "Determine if $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^3$."
Solution: No.

b. Set $A = \begin{bmatrix} 2 & -4 & -2 \\ -5 & 1 & 1 \\ 7 & -5 & -3 \end{bmatrix}$. "Determine if the columns of A span $\mathbb{R}^3$."

c. Define $T(\mathbf{x}) = A\mathbf{x}$. "Determine if T maps $\mathbb{R}^3$ onto $\mathbb{R}^3$."

9. $\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \frac{4}{3}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \frac{7}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 8/3 \\ 4/3 \end{bmatrix} + \begin{bmatrix} 7/3 \\ 14/3 \end{bmatrix}$

10. *Hint:* Construct a "grid" on the x_1x_2-plane determined by $\mathbf{a}_1$ and $\mathbf{a}_2$.

11. A solution set is a line when the system has one free variable. If the coefficient matrix is 2×3, then two of the columns should be pivot columns. For instance, take $\begin{bmatrix} 1 & 2 & * \\ 0 & 3 & * \end{bmatrix}$. Put anything in column 3. The resulting matrix will be in echelon form. Make one row replacement operation on the second row to create a matrix *not* in echelon form, such as $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 2 \end{bmatrix}$.

12. *Hint:* How many free variables are in the equation $A\mathbf{x} = \mathbf{0}$?

13. $E = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

15. a. If the three vectors are linearly independent, then a, c, and f must all be nonzero.

b. The numbers $a, \ldots, f$ can have any values.

16. *Hint:* List the columns from right to left as $\mathbf{v}_1, \ldots, \mathbf{v}_4$.

17. *Hint:* Use Theorem 7.

19. Let M be the line through the origin that is parallel to the line through $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$. Then $\mathbf{v}_2 - \mathbf{v}_1$ and $\mathbf{v}_3 - \mathbf{v}_1$ are both on M. So one of these two vectors is a multiple of the other, say $\mathbf{v}_2 - \mathbf{v}_1 = k(\mathbf{v}_3 - \mathbf{v}_1)$. This equation produces a linear dependence relation: $(k-1)\mathbf{v}_1 + \mathbf{v}_2 - k\mathbf{v}_3 = \mathbf{0}$.

A second solution: A parametric equation of the line is $\mathbf{x} = \mathbf{v}_1 + t(\mathbf{v}_2 - \mathbf{v}_1)$. Since $\mathbf{v}_3$ is on the line, there is some t_0 such that $\mathbf{v}_3 = \mathbf{v}_1 + t_0(\mathbf{v}_2 - \mathbf{v}_1) = (1 - t_0)\mathbf{v}_1 + t_0\mathbf{v}_2$. So $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

21. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **23.** $a = 4/5$ and $b = -3/5$

25. a. The vector lists the number of three-, two-, and one-bedroom apartments provided when x_1 floors of plan A are constructed.

b. $x_1 \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 3 \\ 9 \end{bmatrix}$

c. [**M**] Use 2 floors of plan A and 15 floors of plan B. Or, use 6 floors of plan A, 2 floors of plan B, and 8 floors of plan C. These are the only feasible solutions. There are other mathematical solutions, but they require a negative number of floors of one or two of the plans, which makes no physical sense.

Chapter 2

Section 2.1, page 102

1. $\begin{bmatrix} -4 & 0 & 2 \\ -8 & 10 & -4 \end{bmatrix}$, $\begin{bmatrix} 3 & -5 & 3 \\ -7 & 6 & -7 \end{bmatrix}$, not defined, $\begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$

3. $\begin{bmatrix} -1 & 1 \\ -5 & 5 \end{bmatrix}$, $\begin{bmatrix} 12 & -3 \\ 15 & -6 \end{bmatrix}$

5. a. $A\mathbf{b}_1 = \begin{bmatrix} -7 \\ 7 \\ 12 \end{bmatrix}$, $A\mathbf{b}_2 = \begin{bmatrix} 4 \\ -6 \\ -7 \end{bmatrix}$,

$AB = \begin{bmatrix} -7 & 4 \\ 7 & -6 \\ 12 & -7 \end{bmatrix}$

b. $AB = \begin{bmatrix} -1 \cdot 3 + 2(-2) & -1(-2) + 2 \cdot 1 \\ 5 \cdot 3 + 4(-2) & 5(-2) + 4 \cdot 1 \\ 2 \cdot 3 - 3(-2) & 2(-2) - 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} -7 & 4 \\ 7 & -6 \\ 12 & -7 \end{bmatrix}$

7. 3×7 **9.** $k = 5$

11. $AD = \begin{bmatrix} 2 & 3 & 5 \\ 2 & 6 & 15 \\ 2 & 12 & 25 \end{bmatrix}$, $DA = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 6 & 9 \\ 5 & 20 & 25 \end{bmatrix}$

Right-multiplication (that is, multiplication on the right) by D multiplies each *column* of A by the corresponding diagonal entry of D. Left-multiplication by D multiplies each *row* of A by the corresponding diagonal entry of D. The *Study Guide* tells how to make $AB = BA$, but you should try this yourself before looking there.

13. *Hint:* One of the two matrices is Q.

15. Answer the questions before looking in the *Study Guide*.

17. $\mathbf{b}_1 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$

19. The third column of AB is the sum of the first two columns of AB. Here's why. Write $B = [\,\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3\,]$. By definition, the third column of AB is $A\mathbf{b}_3$. If $\mathbf{b}_3 = \mathbf{b}_1 + \mathbf{b}_2$, then $A\mathbf{b}_3 = A(\mathbf{b}_1 + \mathbf{b}_2) = A\mathbf{b}_1 + A\mathbf{b}_2$, by a property of matrix-vector multiplication.

21. The columns of A are linearly dependent. Why?

23. *Hint:* Suppose $\mathbf{x}$ satisfies $A\mathbf{x} = \mathbf{0}$, and show that $\mathbf{x}$ must be $\mathbf{0}$.

25. *Hint:* Use the results of Exercises 23 and 24, and apply the associative law of multiplication to the product CAD.

27. $\mathbf{u}^T\mathbf{v} = \mathbf{v}^T\mathbf{u} = -2a + 3b - 4c$,

$\mathbf{u}\mathbf{v}^T = \begin{bmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \\ -4a & -4b & -4c \end{bmatrix}$,

$\mathbf{v}\mathbf{u}^T = \begin{bmatrix} -2a & 3a & -4a \\ -2b & 3b & -4b \\ -2c & 3c & -4c \end{bmatrix}$

29. *Hint:* For Theorem 2(b), show that the (i, j)-entry of $A(B + C)$ equals the (i, j)-entry of $AB + AC$.

31. *Hint:* Use the definition of the product $I_m A$ and the fact that $I_m\mathbf{x} = \mathbf{x}$ for $\mathbf{x}$ in $\mathbb{R}^m$.

33. *Hint:* First write the (i, j)-entry of $(AB)^T$, which is the (j, i)-entry of AB. Then, to compute the (i, j)-entry in B^TA^T, use the facts that the entries in row i of B^T are $b_{1i}, \ldots, b_{ni}$, because they come from column i of B, and the entries in column j of A^T are $a_{j1}, \ldots, a_{jn}$, because they come from row j of A.

35. [**M**] The answer here depends on the choice of matrix program. For MATLAB, use the `help` command to read about `zeros`, `ones`, `eye`, and `diag`.

37. [**M**] Display your results and report your conclusions.

39. [**M**] The matrix S "shifts" the entries in a vector (a, b, c, d, e) to yield $(b, c, d, e, 0)$. S^5 is the 5×5 zero matrix. So is S^6.

Section 2.2, page 111

1. $\begin{bmatrix} 2 & -3 \\ -5/2 & 4 \end{bmatrix}$ **3.** $-\frac{1}{5}\begin{bmatrix} -5 & -5 \\ 7 & 8 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix}$

5. $x_1 = 7$ and $x_2 = -9$

7. a and **b**: $\begin{bmatrix} -9 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 11 \\ -5 \end{bmatrix}$, $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 13 \\ -5 \end{bmatrix}$

9. Write out your answers before checking the *Study Guide*.

11. The proof can be modeled after the proof of Theorem 5.

13. $AB = AC \Rightarrow A^{-1}AB = A^{-1}AC \Rightarrow IB = IC \Rightarrow B = C$. No, in general, B and C can be different when A is not invertible. See Exercise 10 in Section 2.1.

15. $D = C^{-1}B^{-1}A^{-1}$. Show that D works.

17. $A = BCB^{-1}$