KEY TERMS AND CONCEPTS	EXAMPLES
The *future value* of a continuous income stream of K dollars per year for N years at interest rate r compounded continuously is $$\int_0^N Ke^{r(N-t)}\,dt.$$	The average is given by $$\frac{1}{20-10}\int_{10}^{20}\left[\frac{x^3}{200}-8x+150\right]dx$$ $$=\frac{1}{10}\left[\frac{x^4}{(4)(200)}-\frac{8}{2}x^2+150x\right]\Bigg\vert_{10}^{20}$$ $$=\frac{1}{10}\left[\frac{x^4}{800}-4x^2+150x\right]\Bigg\vert_{10}^{20}$$ $$=\frac{1}{10}\left[\left[\frac{20^4}{800}-(4)(20)^2+(150)(20)\right]-\left[\frac{10^4}{800}-(4)(10)^2+(150)(10)\right]\right]=48.75.$$ So, if the number of items varies between 10 and 20, the average price will be \$48.75.

Chapter 6 Fundamental Concept Check Exercises

1. What does it mean to antidifferentiate a function?
2. State the formula for $\int h(x)dx$ for each of the following functions.
   (a) $h(x)x^r,\ r \neq -1$ (b) $h(x) = e^{kx}$
   (c) $h(x) = \dfrac{1}{x}$ (d) $h(x) = f(x) + g(x)$
   (e) $h(x) = kf(x)$
3. In the formula $\Delta x = \dfrac{b-a}{n}$, what do a, b, n, and Δx denote?
4. What is a Riemann sum?
5. Give an interpretation of the area under a rate of change function. Give a concrete example.
6. What is a definite integral?
7. What is the difference between a definite integral and an indefinite integral?
8. State the fundamental theorem of calculus.
9. How is $F(x)\Big\vert_a^b$ calculated, and what is it called?
10. Outline a procedure for finding the area of a region bounded by two curves.
11. State the formula for each of the following quantities:
    (a) average value of a function
    (b) consumers' surplus
    (c) future value of an income stream
    (d) volume of a solid of revolution

Chapter 6 Review Exercises

Calculate the following integrals.

1. $\int 3^2\,dx$
2. $\int (x^2 - 3x + 2)dx$
3. $\int \sqrt{x+1}\,dx$
4. $\int \dfrac{2}{x+4}\,dx$
5. $2\int (x^3 + 3x^2 - 1)dx$
6. $\int \sqrt[5]{x+3}\,dx$
7. $\int e^{-x/2}\,dx$
8. $\int \dfrac{5}{\sqrt{x-7}}\,dx$
9. $\int (3x^4 - 4x^3)dx$
10. $\int (2x+3)^7\,dx$
11. $\int \sqrt{4-x}\,dx$
12. $\int \left(\dfrac{5}{x} - \dfrac{x}{5}\right)dx$
13. $\int_{-1}^{1} (x+1)^2\,dx$
14. $\int_0^{1/8} \sqrt[3]{x}\,dx$
15. $\int_{-1}^{2} \sqrt{2x+4}\,dx$
16. $2\int_0^1 \left(\dfrac{2}{x+1} - \dfrac{1}{x+4}\right)dx$

17. $\displaystyle\int_1^2 \frac{4}{x^5}\,dx$

18. $\displaystyle\frac{2}{3}\int_0^8 \sqrt{x+1}\,dx$

19. $\displaystyle\int_1^4 \frac{1}{x^2}\,dx$

20. $\displaystyle\int_3^6 e^{2-(x/3)}\,dx$

21. $\displaystyle\int_0^5 (5+3x)^{-1}\,dx$

22. $\displaystyle\int_{-2}^2 \frac{3}{2e^{3x}}\,dx$

23. $\displaystyle\int_0^{\ln 2} (e^x - e^{-x})dx$

24. $\displaystyle\int_{\ln 2}^{\ln 3} (e^x + e^{-x})dx$

25. $\displaystyle\int_0^{\ln 3} \frac{e^x + e^{-x}}{e^{2x}}\,dx$

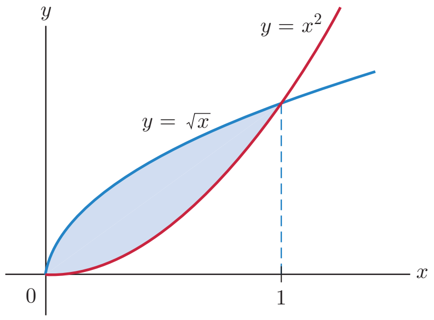
26. $\displaystyle\int_0^1 \frac{3+e^{2x}}{e^x}\,dx$

27. Find the area under the curve $y = (3x-2)^{-3}$ from $x = 1$ to $x = 2$.

28. Find the area under the curve $y = 1 + \sqrt{x}$ from $x = 1$ to $x = 9$.

In Exercises 29–36, find the area of the shaded region.

29.



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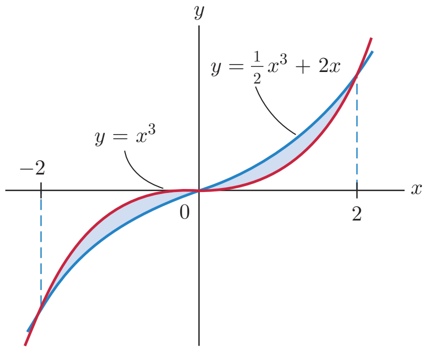

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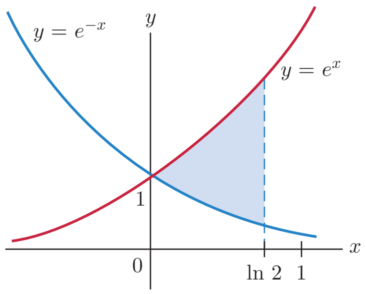

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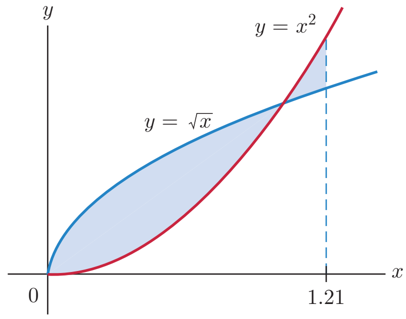

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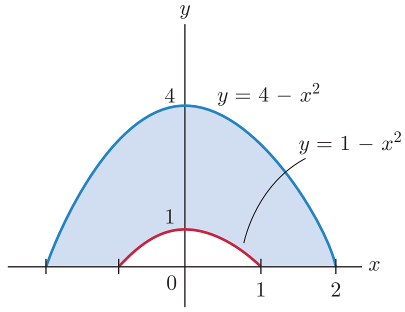

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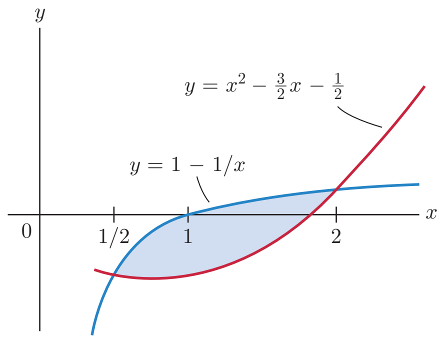

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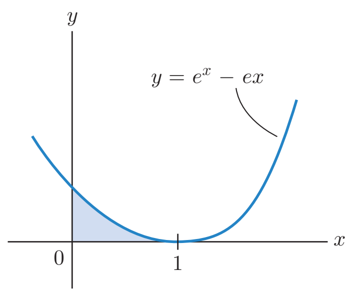

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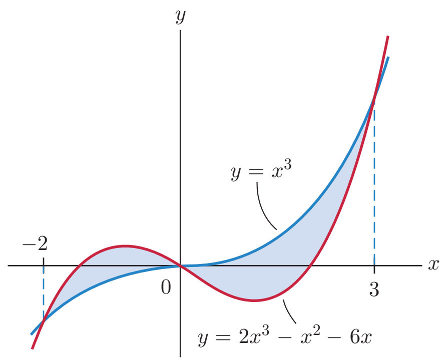

37. Find the area of the region bounded by the curves $y = x^3 - 3x + 1$ and $y = x + 1$.

38. Find the area of the region between the curves $y = 2x^2 + x$ and $y = x^2 + 2$ from $x = 0$ to $x = 2$.

39. Find the function $f(x)$ for which $f'(x) = (x - 5)^2$, $f(8) = 2$.

40. Find the function $f(x)$ for which $f'(x) = e^{-5x}$, $f(0) = 1$.

41. Describe all solutions of the following differential equations, where y represents a function of t.

(a) $y' = 4t$ **(b)** $y' = 4y$ **(c)** $y' = e^{4t}$

42. Let k be a constant, and let $y = f(t)$ be a function such that $y' = kty$. Show that $y = Ce^{kt^2/2}$, for some constant C. [*Hint:* Use the product rule to evaluate $\frac{d}{dt}[f(t)e^{-kt^2/2}]$, and then apply Theorem II of Section 6.1.]

43. An airplane tire plant finds that its marginal cost of producing tires is $.04x + \$150$ at a production level of x tires per day. If fixed costs are \$500 per day, find the cost of producing x tires per day.

44. If the marginal revenue function for a company is $400 - 3x^2$, find the additional revenue received from doubling production if 10 units are currently being produced.

45. A drug is injected into a patient at the rate of $f(t)$ cubic centimeters per minute at time t. What does the area under the graph of $y = f(t)$ from $t = 0$ to $t = 4$ represent?

46. A rock thrown straight up into the air has a velocity of $v(t) = -9.8t + 20$ meters per second after t seconds.

(a) Determine the distance the rock travels during the first 2 seconds.

(b) Represent the answer to part (a) as an area.

47. Use a Riemann sum with $n = 4$ and left endpoints to estimate the area under the graph in Fig. 1 for $0 \le x \le 2$.

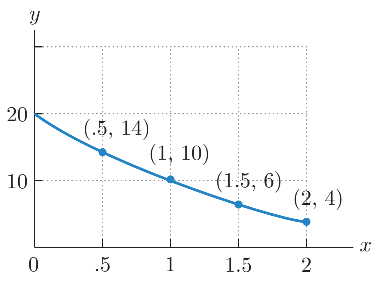

Figure 1

48. Redo Exercise 47 using right endpoints.

49. Use a Riemann sum with $n = 2$ and midpoints to estimate the area under the graph of

$$f(x) = \frac{1}{x+2}$$

on the interval $0 \le x \le 2$. Then, use a definite integral to find the exact value of the area to five decimal places.

50. Use a Riemann sum with $n = 5$ and midpoints to estimate the area under the graph of $f(x) = e^{2x}$ on the interval $0 \le x \le 1$. Then, use a definite integral to find the exact value of the area to five decimal places.

51. Find the consumers' surplus for the demand curve $p = \sqrt{25 - .04x}$ at the sales level $x = 400$.

52. Three thousand dollars is deposited in the bank at 4% interest compounded continuously. What will be the average value of the money in the account during the next 10 years?

53. Find the average value of $f(x) = 1/x^3$ from $x = \frac{1}{3}$ to $x = \frac{1}{2}$.

54. Suppose that the interval $0 \le x \le 1$ is divided into 100 subintervals with a width of $\Delta x = .01$. Show that the sum

$$[3e^{-.01}]\,\Delta x + [3e^{-.02}]\,\Delta x + [3e^{-.03}]\,\Delta x + \cdots + [3e^{-1}]\,\Delta x$$

is close to $3(1 - e^{-1})$.

55. In Fig. 2, three regions are labeled with their areas. Determine $\int_a^c f(x)dx$ and determine $\int_a^d f(x)dx$.

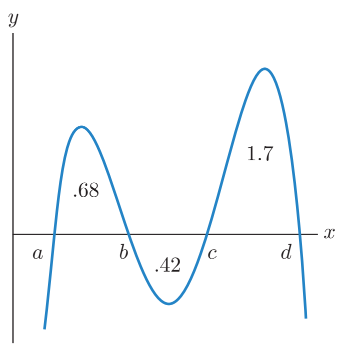

Figure 2

56. Find the volume of the solid of revolution generated by revolving about the x-axis the region under the curve $y = 1 - x^2$ from $x = 0$ to $x = 1$.

57. A store has an inventory of Q units of a certain product at time $t = 0$. The store sells the product at the steady rate of Q/A units per week and exhausts the inventory in A weeks.

(a) Find a formula $f(t)$ for the amount of product in inventory at time t.

(b) Find the average inventory level during the period $0 \le t \le A$.

58. A retail store sells a certain product at the rate of $g(t)$ units per week at time t, where $g(t) = rt$. At time $t = 0$, the store has Q units of the product in inventory.

(a) Find a formula $f(t)$ for the amount of product in inventory at time t.

(b) Determine the value of r in part (a) such that the inventory is exhausted in A weeks.

(c) Using $f(t)$, with r as in part (b), find the average inventory level during the period $0 \le t \le A$.

59. Let x be any positive number, and define $g(x)$ to be the number determined by the definite integral

$$g(x) = \int_0^x \frac{1}{1+t^2}\, dt.$$

(a) Give a geometric interpretation of the number $g(3)$.

(b) Find the derivative $g'(x)$.

60. For each number x satisfying $-1 \le x \le 1$, define $h(x)$ by

$$h(x) = \int_{-1}^x \sqrt{1-t^2}\, dt.$$

(a) Give a geometric interpretation of the values $h(0)$ and $h(1)$.

(b) Find the derivative $h'(x)$.

61. Suppose that the interval $0 \le t \le 3$ is divided into 1000 subintervals of width Δt. Let $t_1, t_2, \ldots, t_{1000}$ denote the right endpoints of these subintervals. If we need to estimate the sum

$$5000e^{-.1t_1}\Delta t + 5000e^{-.1t_2}\Delta t + \cdots + 5000e^{-.1t_{1000}}\Delta t,$$

show that this sum is close to 13,000. [*Note:* A sum such as this would arise if we wanted to compute the present value of a continuous stream of income of \$5000 per year for 3 years, with interest compounded continuously at 10%.]

62. What number does

$$\left[e^0 + e^{1/n} + e^{2/n} + e^{3/n} + \cdots + e^{(n-1)/n}\right] \cdot \frac{1}{n}$$

approach as n gets very large?

63. What number does the sum

$$\left[1^3 + \left(1+\frac{1}{n}\right)^3 + \left(1+\frac{2}{n}\right)^3 + \left(1+\frac{3}{n}\right)^3 + \cdots + \left(1+\frac{n-1}{n}\right)^3\right] \cdot \frac{1}{n}$$

approach as n gets very large?

64. In Fig. 3, the rectangle has the same area as the region under the graph of $f(x)$. What is the average value of $f(x)$ on the interval $2 \le x \le 6$?

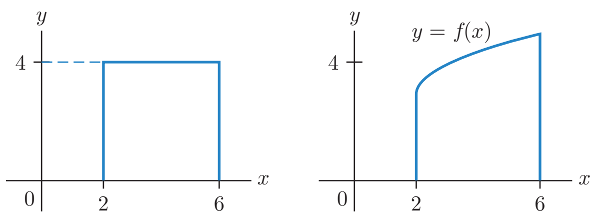

Figure 3

65. *True or false*: If $3 \le f(x) \le 4$ whenever $0 \le x \le 5$, then $3 \le \frac{1}{5}\int_0^5 f(x)dx \le 4$.

66. Suppose that water is flowing into a tank at a rate of $r(t)$ gallons per hour, where the rate depends on the time t according to the formula

$$r(t) = 20 - 4t, \qquad 0 \le t \le 5.$$

(a) Consider a brief period of time, say from t_1 to t_2. The length of this time period is $\Delta t = t_2 - t_1$. During this period the rate of flow does not change much and is approximately $20 - 4t_1$ (the rate at the beginning of the brief time interval). Approximately how much water flows into the tank during the time from t_1 to t_2?

(b) Explain why the total amount of water added to the tank during the time interval from $t = 0$ to $t = 5$ is given by $\int_0^5 r(t)dt$.

67. The annual world rate of water use t years after 1960, for $t \le 35$, was approximately $860e^{.04t}$ cubic kilometers per year. How much water was used between 1960 and 1995?

68. If money is deposited steadily in a savings account at the rate of \$4500 per year, determine the balance at the end of 1 year if the account pays 9% interest compounded continuously.

69. Find a function $f(x)$ whose graph goes through the point $(1, 1)$ and whose slope at any point $(x, f(x))$ is $3x^2 - 2x + 1$.

70. For what value of a is the shaded area in Fig. 4 equal to 1?

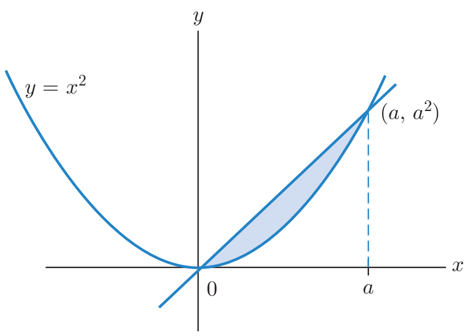

Figure 4

71. Show that for any positive number b we have

$$\int_0^{b^2} \sqrt{x}\, dx + \int_0^b x^2\, dx = b^3.$$

72. Generalize the result of Exercise 71 as follows: Let n be a positive integer. Show that for any positive number b we have

$$\int_0^{b^n} \sqrt[n]{x}\, dx + \int_0^b x^n\, dx = b^{n+1}.$$

73. Show that

$$\int_0^1 (\sqrt{x} - x^2)dx = \frac{1}{3}.$$

74. Generalize the result of Exercise 73 as follows: Let n be a positive integer. Show that

$$\int_0^1 (\sqrt[n]{x} - x^n)dx = \frac{n-1}{n+1}.$$