Check Your Understanding 6.4

1. Find the area between the curves $y = x + 3$ and $y = \frac{1}{2}x^2 + x - 7$ from $x = -2$ to $x = 1$.

2. A company plans to increase its production from 10 to 15 units per day. The present marginal cost function is $MC_1(x) = x^2 - 20x + 108$. By redesigning the production process and purchasing new equipment, the company can change the marginal cost function to $MC_2(x) = \frac{1}{2}x^2 - 12x + 75$. Determine the area between the graphs of the two marginal cost curves from $x = 10$ to $x = 15$. Interpret this area in economic terms.

EXERCISES 6.4

1. Write down a definite integral or sum of definite integrals that gives the area of the shaded portions in Fig. 21.

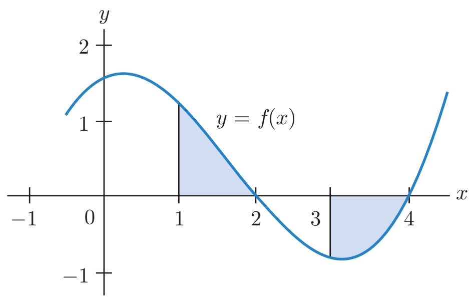

Figure 21

2. Write down a definite integral or sum of definite integrals that gives the area of the shaded portions in Fig. 22.

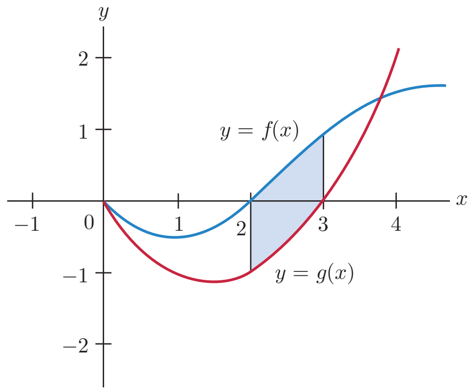

Figure 22

3. Shade the portion of Fig. 23 whose area is given by the integral

$$\int_0^2 [f(x) - g(x)]dx + \int_2^4 [h(x) - g(x)]dx.$$

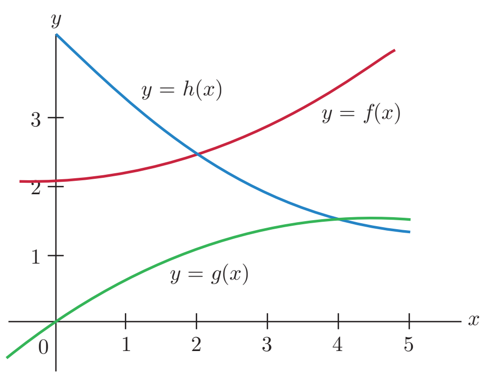

Figure 23

4. Shade the portion of Fig. 24 whose area is given by the integral

$$\int_0^1 [f(x) - g(x)] + \int_1^2 [g(x) - f(x)]dx.$$

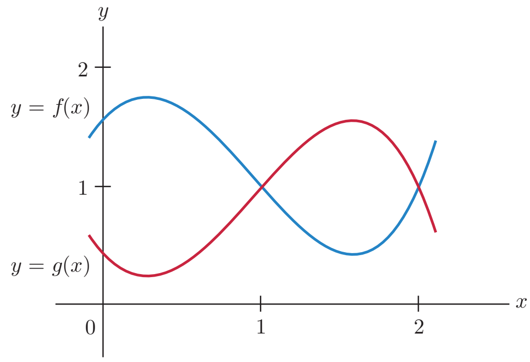

Figure 24

5. Let $f(x)$ be the function pictured in Fig. 25. Determine whether $\int_0^7 f(x)dx$ is positive, negative, or zero.

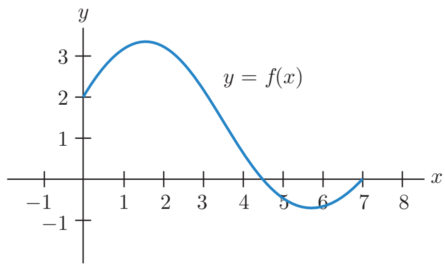

Figure 25

6. Let $g(x)$ be the function pictured in Fig. 26. Determine whether $\int_0^7 g(x)dx$ is positive, negative, or zero.

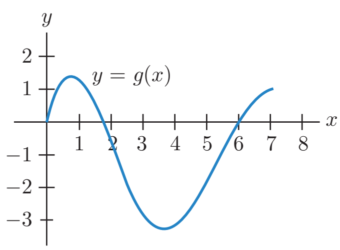

Figure 26

Find the area of the region between the curve and the x-axis.

7. $f(x) = 1 - x^2$, from -2 to 2

8. $f(x) = x(x^2 - 1)$, from -1 to 1.

9. $f(x) = x^2 - 2x - 3$, from 0 to 2.

10. $f(x) = x^2 + 6x + 5$ from 0 to 1.

11. $f(x) = e^x - 3$ from 0 to $\ln 3$

12. $f(x) = e^{-x} + 2$ from -1 to 2

Find the area of the region between the curves.

13. $y = 2x^2$ and $y = 8$ (a horizontal line) from $x = -2$ to $x = 2$

14. $y = x^2 + 1$ and $y = -x^2 - 1$ from $x = -1$ to $x = 1$

15. $y = x^2 - 6x + 12$ and $y = 1$ from $x = 0$ to $x = 4$

16. $y = x(2 - x)$ and $y = 2$ from $x = 0$ to $x = 2$

17. $y = e^x$ and $y = \frac{1}{x^2}$ from $x = 1$ to $x = 2$

18. $y = e^{2x}$ and $y = 1 - x$ from $x = 0$ to $x = 1$

Find the area of the region bounded by the curves.

19. $y = x^2$ and $y = x$

20. $y = 4x(1 - x)$ and $y = \frac{3}{4}$

21. $y = -x^2 + 6x - 5$ and $y = 2x - 5$

22. $y = x^2 - 1$ and $y = 3$

23. $y = x(x^2 - 1)$ and the x-axis

24. $y = x^3$ and $y = 2x^2$

25. $y = 8x^2$ and $y = \sqrt{x}$

26. $y = \frac{4}{x}$ and $y = 5 - x$

27. Find the area of the region between $y = x^2 - 3x$ and the x-axis

(a) from $x = 0$ to $x = 3$,

(b) from $x = 0$ to $x = 4$,

(c) from $x = -2$ to $x = 3$.

28. Find the area of the region between $y = x^2$ and $y = 1/x^2$

(a) from $x = 1$ to $x = 4$,

(b) from $x = \frac{1}{2}$ to $x = 4$.

29. Find the area in Fig. 27 of the region bounded by $y = 1/x^2$, $y = x$, and $y = 8x$, for $x \geq 0$.

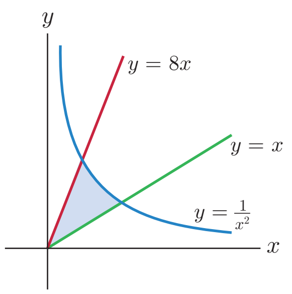

Figure 27

30. Find the area of the region bounded by $y = 1/x$, $y = 4x$, and $y = x/2$, for $x \geq 0$. (The region resembles the shaded region in Exercise 29.)

31. **Height of a Helicopter** A helicopter is rising straight up in the air. Its velocity at time t is $v(t) = 2t + 1$ feet per second.

(a) How high does the helicopter rise during the first 5 seconds?

(b) Represent the answer to part (a) as an area.

32. **Assembly Line Production** After t hours of operation, an assembly line is producing lawn mowers at the rate of $r(t) = 21 - \frac{4}{5}t$ mowers per hour.

(a) How many mowers are produced during the time from $t = 2$ to $t = 5$ hours?

(b) Represent the answer to part (a) as an area.

33. **Cost** Suppose that the marginal cost function of a handbag manufacturer is $C'(x) = \frac{3}{32}x^2 - x + 200$ dollars per unit at production level x (where x is measured in units of 100 handbags).

(a) Find the total cost of producing 6 additional units if 2 units are currently being produced.

(b) Describe the answer to part (a) as an area. (Give a written description rather than a sketch.)

34. **Profit** Suppose that the marginal profit function for a company is $P'(x) = 100 + 50x - 3x^2$ at production level x.

(a) Find the extra profit earned from the sale of 3 additional units if 5 units are currently being produced.

(b) Describe the answer to part (a) as an area. (Do not make a sketch.)

35. **Marginal Profit** Let $MP(x)$ be a company's marginal profit at production level x. Give an economic interpretation of the number $\int_{44}^{48} MP(x)dx$.

36. **Marginal Profit** Let $MC(x)$ be a company's marginal cost at production level x. Give an economic interpretation of the number $\int_0^{100} MC(x)dx$. (*Note:* At any production level, the total cost equals the fixed cost plus the total variable cost.)

37. **Heat Diffusion** Some food is placed in a freezer. After t hours, the temperature of the food is dropping at the rate of $r(t)$ degrees Fahrenheit per hour, where $r(t) = 12 + 4/(t + 3)^2$.

(a) Compute the area under the graph of $y = r(t)$ over the interval $0 \leq t \leq 2$.

(b) What does the area in part (a) represent?

38. **Velocity** Suppose that the velocity of a car at time t is $v(t) = 40 + 8/(t + 1)^2$ kilometers per hour.

(a) Compute the area under the velocity curve from $t = 1$ to $t = 9$.

(b) What does the area in part (a) represent?

39. **Deforestation and Fuel Wood** Deforestation is one of the major problems facing sub-Saharan Africa. Although the clearing of land for farming has been the major cause, the steadily increasing demand for fuel wood has also become a significant factor. Figure 28 summarizes

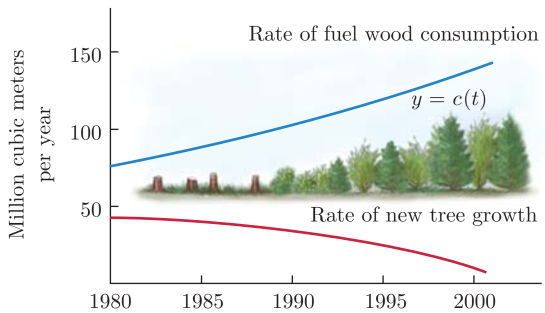

Figure 28

projections of the World Bank. The rate of fuel wood consumption (in millions of cubic meters per year) in the Sudan t years after 1980 is given approximately by the function $c(t) = 76.2e^{.03t}$. Determine the amount of fuel wood consumed from 1980 to 2000.

40. Refer to Exercise 39. The rate of new tree growth (in millions of cubic meters per year) in the Sudan t years after 1980 is given approximately by the function $g(t) = 50 - 6.03e^{.09t}$. Set up the definite integral giving the amount of depletion of the forests due to the excess of fuel wood consumption over new growth from 1980 to 2000.

41. **World Oil Consumption** Refer to the oil-consumption data in Example 10. Suppose that in 1970 the growth constant for the annual rate of oil consumption had been held to .04. What effect would this action have had on oil consumption from 1970 to 1974?

42. **Profit and Area** The marginal profit for a certain company is $MP_1(x) = -x^2 + 14x - 24$. The company expects the daily production level to rise from $x = 6$ to $x = 8$ units. The management is considering a plan that would have the effect of changing the marginal profit to $M_2(x) = -x^2 + 12x - 20$. Should the company adopt the plan? Determine the area between the graphs of the two marginal profit functions from $x = 6$ to $x = 8$. Interpret this area in economic terms.

43. **Velocity and Distance** Two rockets are fired simultaneously straight up into the air. Their velocities (in meters per second) are $v_1(t)$ and $v_2(t)$, and $v_1(t) \geq v_2(t)$ for $t \geq 0$. Let A denote the area of the region between the graphs of $y = v_1(t)$ and $y = v_2(t)$ for $0 \leq t \leq 10$. What physical interpretation may be given to the value of A?

44. **Distance Traveled** Cars A and B start at the same place and travel in the same direction, with velocities after t hours given by the functions $v_A(t)$ and $v_B(t)$ in Fig. 29.

(a) What does the area between the two curves from $t = 0$ to $t = 1$ represent?

(b) At what time will the distance between the cars be greatest?

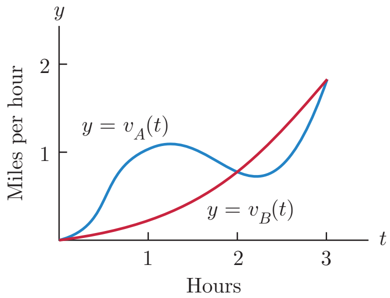

Figure 29

45. **Displacement versus Distance Traveled** The velocity of an object moving along a line is given by $v(t) = 2t^2 - 3t + 1$ feet per second.

(a) Find the displacement of the object as t varies in the interval $0 \leq t \leq 3$.

(b) Find the total distance traveled by the object during the interval of time $0 \leq t \leq 3$.

46. **Displacement versus Distance Traveled** The velocity of an object moving along a line is given by $v(t) = t^2 + t - 2$ feet per second.

(a) Find the displacement of the object as t varies in the interval $0 \leq t \leq 3$. Interpret this displacement using area under the graph of $v(t)$.

(b) Find the total distance traveled by the object during the interval of time $0 \leq t \leq 3$. Interpret this distance as an area.

Technology Exercises

In Exercises 47–50, use a graphing utility to find the intersection points of the curves, and then use the utility to find the area of the region bounded by the curves.

47. $y = e^x$, $y = 4x + 1$

48. $y = 5 - (x - 2)^2$, $y = e^x$

49. $y = \sqrt{x + 1}$, $y = (x - 1)^2$

50. $y = 1/x$, $y = 3 - x$

Solutions to Check Your Understanding 6.4

1. First, graph the two curves, as shown in Fig. 30. The curve $y = x + 3$ lies on top. So, the area between the curves is

$$\int_{-2}^{1} \left[(x + 3) - \left(\tfrac{1}{2}x^2 + x - 7\right)\right] dx$$

$$= \int_{-2}^{1} \left(-\tfrac{1}{2}x^2 + 10\right) dx$$

$$= \left(-\tfrac{1}{6}x^3 + 10x\right)\Big|_{-2}^{1} = 28.5.$$

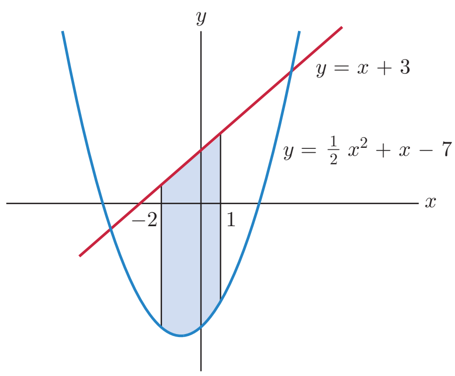

Figure 30