INCORPORATING TECHNOLOGY

Computing a Definite Integral The definite integral in Example 7 is evaluated in Fig. 4. To do this evaluation, select MATH 9. Complete the integral, as shown in Fig. 4. Then press ENTER.

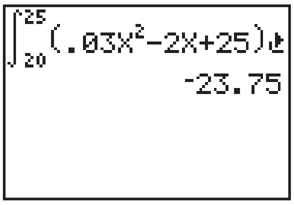

Figure 4

Check Your Understanding 6.2

1. Evaluate $\displaystyle\int_0^1 \frac{e^{2x}-1}{e^x}\,dx$.

EXERCISES 6.2

In Exercises 1–14, evaluate the given integral.

1. $\displaystyle\int_0^1 \left(2x - \frac{3}{4}\right)dx$

2. $\displaystyle\int_{-1}^2 7\left(\frac{x^2}{3} - 3x\right)dx$

3. $\displaystyle\int_1^4 (3\sqrt{t} + 4t)\,dt$

4. $\displaystyle\int_1^9 \frac{3}{\sqrt{x}}\,dx$

5. $\displaystyle\int_1^4 \left(x - \frac{3}{4x^2}\right)dx$

6. $\displaystyle\int_1^8 (-x + 11\sqrt[3]{x})\,dx$

7. $\displaystyle\int_1^3 \left(\frac{3x - 2x^3 + 4x^5}{4x^7}\right)dx$

8. $\displaystyle\int_2^6 \left(\frac{3x + \sqrt{x}}{4x^3}\right)dx$

9. $\displaystyle\int_{-1}^0 (3e^{3t} + t)\,dt$

10. $\displaystyle\int_{-2}^2 (e^t + e^{-t})\,dt$

11. $\displaystyle\int_1^2 \frac{2}{x}\,dx$

12. $\displaystyle\int_{-2}^{-1} \frac{1+x}{x}\,dx$

13. $\displaystyle\int_0^1 \frac{e^x + e^{2x} - 7}{e^{3x}}\,dx$

14. $\displaystyle\int_0^{\ln 2} \frac{e^x + e^{-x}}{2}\,dx$

15. Given $\displaystyle\int_0^1 f(x)\,dx = 3.5$ and $\displaystyle\int_1^4 f(x)\,dx = 5$, find $\displaystyle\int_0^4 f(x)\,dx$.

16. Given $\displaystyle\int_{-1}^1 f(x)\,dx = 0$ and $\displaystyle\int_{-1}^{10} f(x)\,dx = 4$, find $\displaystyle\int_1^{10} f(x)\,dx$.

17. Given $\displaystyle\int_1^3 f(x)\,dx = 3$ and $\displaystyle\int_1^3 g(x)\,dx = -1$, find $\displaystyle\int_1^3 (2f(x) - 3g(x))\,dx$.

18. Given $\displaystyle\int_{-.5}^3 f(x)\,dx = 0$ and $\displaystyle\int_{-.5}^3 (2g(x) + f(x))\,dx = -4$, find $\displaystyle\int_{-.5}^3 g(x)\,dx$.

In Exercises 19–22, combine the integrals into one integral, then evaluate the integral.

19. $\displaystyle 2\int_1^2 \left(3x + \frac{1}{2}x^2 - x^3\right)dx + 3\int_1^2 (x^2 - 2x + 7)\,dx$

20. $\displaystyle\int_{.5}^{1.5} \left(-2x - \frac{x^3}{3}\right)dx + 2\int_{.5}^{1.5} (\sqrt{x} + x)\,dx$

21. $\displaystyle\int_{-1}^0 (x^3 + x^2)\,dx + \int_0^1 (x^3 + x^2)\,dx$

22. $\displaystyle\int_0^1 (7x + 4)\,dx + \int_1^2 (7x + 5)\,dx$

23. Given $f'(x) = -2x + 3$, compute $f(3) - f(1)$. [*Hint:* Use (8).]

24. Given $f'(x) = 73$, compute $f(4) - f(2)$. [*Hint:* Use (8).]

25. Given $f'(t) = -.5t + e^{-2t}$, compute $f(1) - f(-1)$. [*Hint:* Use (8).]

26. Given $f'(t) = -12t - \dfrac{1}{e^t}$, compute $f(3) - f(0)$. [*Hint:* Use (8).]

27. Refer to Fig. 5 and evaluate $\displaystyle\int_0^2 f(x)\,dx$.

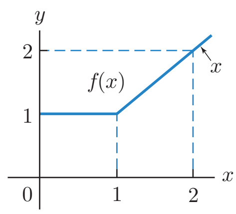

Figure 5

28. Refer to Fig. 6 and evaluate $\displaystyle\int_0^3 f(x)\,dx$.

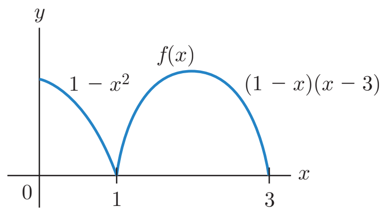

Figure 6

29. Refer to Fig. 7 and evaluate $\displaystyle\int_{-1}^{1} f(t)dt$.

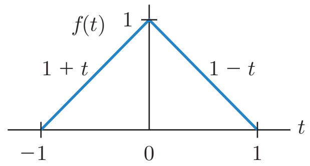

Figure 7

30. Refer to Fig. 8 and evaluate $\displaystyle\int_{-1}^{2} f(t)dt$.

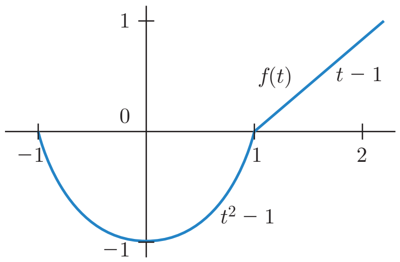

Figure 8

31. Net Change in Position A rock is dropped from the top of a 400-foot cliff. Its velocity at time t seconds is $v(t) = -32t$ feet per second. Find the displacement of the rock during the time interval $2 \le t \le 4$.

32. Net Change in Position The velocity at time t seconds of a ball thrown up into the air is $v(t) = -32t + 75$ feet per second.

(a) Find the displacement of the ball during the time interval $0 \le t \le 3$.

(b) Given that the initial position of the ball is $s(0) = 6$ feet, use (a) to determine its position at time $t = 3$.

33. Net Change in Position The velocity at time t seconds of a ball thrown up into the air is $v(t) = -32t + 75$ feet per second.

(a) Compute the displacement of the ball during the time interval $1 \le t \le 3$. Is the position of the ball at time $t = 3$ higher than its position at time $t = 1$? Justify your answer.

(b) Repeat part (a) using the time interval $1 \le t \le 5$.

34. Velocity of a Skydiver The velocity of a skydiver at time t seconds is $v(t) = 45 - 45e^{-.2t}$ meters per second. Find the distance traveled by the skydiver the first 9 seconds.

35. Net Change in Cost A company's marginal cost function is $.1x^2 - x + 12$ dollars, where x denotes the number of units produced in 1 day.

(a) Determine the increase in cost if the production level is raised from $x = 1$ to $x = 3$ units.

(b) If $C(1) = 15$, determine $C(3)$ using your answer in (a).

36. Cost Increase A company's marginal cost function is given by $C'(x) = 32 + \frac{x}{20}$, where x denotes the number of items produced in 1 day and $C(x)$ is in thousands of dollars. Determine the increase in cost if the company goes from a production level of 15 to 20 items per day.

37. Net Increase of an Investment An investment grew at an exponential rate $R(t) = 700e^{.07t} + 1000$, where t is in years and $R(t)$ is in dollars per year. Approximate the net increase in value of the investment after the first 10 years (as t varies from 0 to 10).

38. Depreciation of Real Estate A property with an appraised value of \$200,000 in 2008 is depreciating at the rate $R(t) = -8e^{-.04t}$, where t is in years since 2008 and $R(t)$ is in thousands of dollars per year. Estimate the loss in value of the property between 2008 and 2014 (as t varies from 0 to 6).

39. Population Model with Emigration The rate of change of a population with emigration is given by $P'(t) = \frac{7}{300}e^{t/25} - \frac{1}{80}e^{t/16}$, where $P(t)$ is the population in millions, t years after the year 2000.

(a) Estimate the change in population as t varies from 2000 to 2010.

(b) Estimate the change in population as t varies from 2010 to 2040. Compare and explain your answers in (a) and (b).

40. Paying Down a Mortgage You took a \$200,000 home mortgage at an annual interest rate of 3%. Suppose that the loan is amortized over a period of 30 years, and let $P(t)$ denote the amount of money (in thousands of dollars) that you owe on the loan after t years. A reasonable estimate of the rate of change of P is given by $P'(t) = -4.1107e^{0.03t}$.

(a) Approximate the net change in P after 20 years.

(b) What is the amount of money owed on the loan after 20 years?

(c) Verify that the loan is paid off in 30 years by computing the net change in P after 30 years.

41. Mortgage Using the data from the previous exercise, find $P(t)$. [*Hint:* $P(0) = 200$.]

42. Radioactive Decay A sample of radioactive material with decay constant .1 is decaying at a rate $R(t) = -e^{-.1t}$ grams per year. How many grams of this material decayed after the first 10 years?

43. Saline Solution A saline solution is being flushed with fresh water in such a way that salt is eliminated at the rate $r(t) = -(t + \frac{1}{2})$ grams per minute. Find the amount of salt that is eliminated during the first 2 minutes.

44. Level of Water in a Tank A conical-shaped tank is being drained. The height of the water level in the tank is decreasing at the rate $h'(t) = -\frac{t}{2}$ inches per minute. Find the decrease in the depth of the water in the tank during the time interval $2 \le t \le 4$.

Solution to Check Your Understanding 6.2

1. Start by simplifying

$$\frac{e^{2x}-1}{e^x} = \frac{e^{2x}}{e^x} - \frac{1}{e^x} = e^x - e^{-x}.$$

An antiderivative of e^x is e^x, and an antiderivative of e^{-x} is $-e^{-x}$. Hence, an antiderivative of $e^x - e^{-x}$ is $e^x - (-e^{-x})$, and so,

$$\int_0^1 \frac{e^{2x}-1}{e^x}dx = \int_0^1 (e^x - e^{-x})dx$$
$$= (e^x + e^{-x})\Big|_0^1$$
$$= (e + e^{-1}) - (1+1) = e + 1/e - 2 \approx 1.09.$$

6.3 The Definite Integral and Area under a Graph

This section and the next reveal the important connection between definite integrals and areas of regions under curves. We start by defining one type of region that we will be considering.

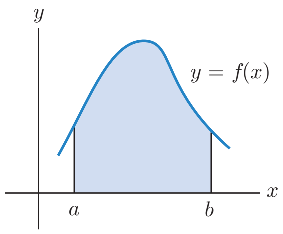

Figure 1 Area under a graph.

Area under a Graph If $f(x)$ is a continuous nonnegative function on the interval $a \le x \le b$, we refer to the area of the region shown in Fig. 1 as the *area under the graph of* $f(x)$ *from* a *to* b, or the *area bounded by the graph of* $f(x)$, *the* x-*axis, and the (vertical) lines* $x = a$ *and* $x = b$.

In this section we solve the area problem, which consists of finding the area of the region under the graph of a continuous function $f(x)$ from a to b as illustrated in Fig. 1.

Many areas of this type are easy to compute with simple geometric constructions.

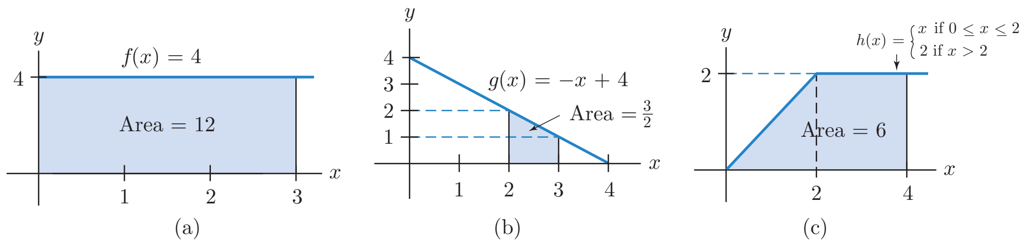

Figure 2 Areas under graphs.

In Fig. 2(a), the shaded rectangular region is under the graph of the constant function $f(x) = 4$ from $x = 0$ to $x = 3$. Its area is $3 \times 4 = 12$.

In Fig. 2(b), the shaded region under the graph of the function $g(x) = -x + 4$ from $x = 2$ to $x = 3$ is a trapezoid that consists of a right triangle on top of a square. Its total area is $\frac{1}{2} + 1 = \frac{3}{2}$.

In Fig. 2(c), we shaded a region under the graph of a "ramp function." Its area is also the sum of the areas of a triangle plus a rectangle and is equal to $2 + 4 = 6$.

In all three examples in Fig. 2, the top boundary of the region consists of line segments. The areas in these cases can be computed with simple geometric constructions. The computation of the area such as graphed in Fig. 1 is not a trivial matter when the top boundary of the region is curved. Consider, for example, the region under the graph of the parabola $f(x) = x^2$ from $x = 0$ to $x = 1$ (Fig. 3). It is not hard to see that this area is less than $1/2$. But what is the exact value of the area? Obviously, the answer cannot be derived from simple geometric formulas. We will show how to solve this area problem using important techniques based on approximations with rectangles. These same techniques will also be used to establish the following fundamental result in calculus, which provides the solution to the area problem.

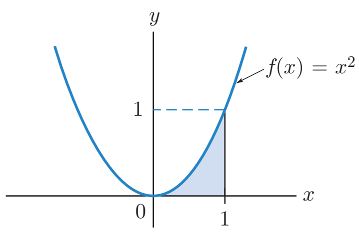

Figure 3 Area under the parabola.