Check Your Understanding 6.1

1. Determine the following:

(a) $\int t^{7/2}\,dt$ (b) $\int \left(\frac{x^3}{3} + \frac{3}{x^3} + \frac{3}{x}\right)dx$

EXERCISES 6.1

Find all antiderivatives of each following function:

1. $f(x) = x$ **2.** $f(x) = 9x^8$ **3.** $f(x) = e^{3x}$

4. $f(x) = e^{-3x}$ **5.** $f(x) = 3$ **6.** $f(x) = -4x$

Determine the following:

7. $\int 4x^3\,dx$ **8.** $\int \frac{x}{3}\,dx$ **9.** $\int 7\,dx$

10. $\int k^2\,dx$ (k a constant)

11. $\int \frac{x}{c}\,dx$ (c a constant $\neq 0$)

12. $\int x \cdot x^2\,dx$ **13.** $\int \left(\frac{2}{x} + \frac{x}{2}\right)dx$

14. $\int \frac{1}{7x}\,dx$ **15.** $\int x\sqrt{x}\,dx$

16. $\int \left(\frac{2}{\sqrt{x}} + 2\sqrt{x}\right)dx$ **17.** $\int \left(x - 2x^2 + \frac{1}{3x}\right)dx$

18. $\int \left(\frac{7}{2x^3} - \sqrt[3]{x}\right)dx$ **19.** $\int 3e^{-2x}\,dx$

20. $\int e^{-x}\,dx$ **21.** $\int e\,dx$

22. $\int \frac{7}{2e^{2x}}\,dx$ **23.** $\int -2(e^{2x}+1)dx$

24. $\int \left(-3e^{-x} + 2x - \frac{e^{.5x}}{2}\right)dx$

In Exercises 25–36, find the value of k that makes the antidifferentiation formula true. [*Note:* You can check your answer without looking in the answer section. How?]

25. $\int 5e^{-2t}\,dt = ke^{-2t} + C$

26. $\int 3e^{t/10}\,dt = ke^{t/10} + C$

27. $\int 2e^{4x-1}\,dx = ke^{4x-1} + C$

28. $\int \frac{4}{e^{3x+1}}\,dx = \frac{k}{e^{3x+1}} + C$

29. $\int (5x-7)^{-2}\,dx = k(5x-7)^{-1} + C$

30. $\int \sqrt{x+1}\,dx = k(x+1)^{3/2} + C$

31. $\int (4-x)^{-1}\,dx = k\ln|4-x| + C$

32. $\int \frac{7}{(8-x)^4}\,dx = \frac{k}{(8-x)^3} + C$

33. $\int (3x+2)^4\,dx = k(3x+2)^5 + C$

34. $\int (2x-1)^3\,dx = k(2x-1)^4 + C$

35. $\int \frac{3}{2+x}\,dx = k\ln|2+x| + C$

36. $\int \frac{5}{2-3x}\,dx = k\ln|2-3x| + C$

Find all functions $f(t)$ with the following property:

37. $f'(t) = t^{3/2}$ **38.** $f'(t) = \frac{4}{6+t}$

39. $f'(t) = 0$ **40.** $f'(t) = t^2 - 5t - 7$

Find all functions $f(x)$ with the following properties:

41. $f'(x) = .5e^{-.2x}$, $f(0) = 0$

42. $f'(x) = 2x - e^{-x}$, $f(0) = 1$

43. $f'(x) = x$, $f(0) = 3$

44. $f'(x) = 8x^{1/3}$, $f(1) = 4$

45. $f'(x) = \sqrt{x} + 1$, $f(4) = 0$

46. $f'(x) = x^2 + \sqrt{x}$, $f(1) = 3$

47. Figure 4 shows the graphs of several functions $f(x)$ for which $f'(x) = \frac{2}{x}$. Find the expression for the function $f(x)$ whose graph passes through $(1, 2)$.

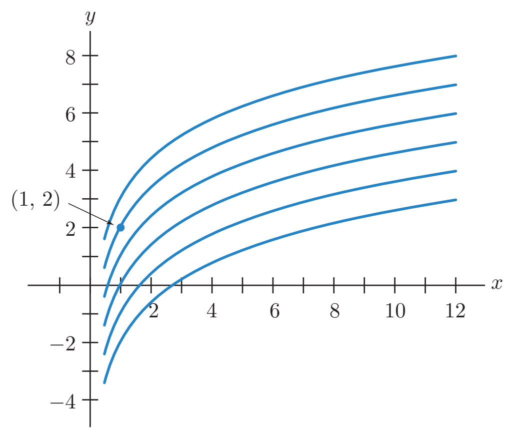

Figure 4

48. Figure 5 shows the graphs of several functions $f(x)$ for which $f'(x) = \frac{1}{3}$. Find the expression for the function $f(x)$ whose graph passes through $(6, 3)$.

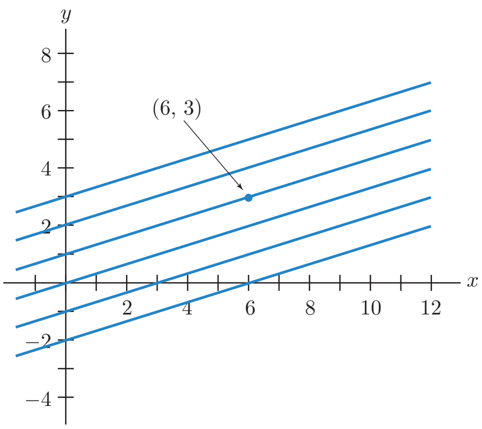

Figure 5

49. Which of the following is $\int \ln x\, dx$?

(a) $\frac{1}{x} + C$ **(b)** $x \cdot \ln x - x + C$

(c) $\frac{1}{2} \cdot (\ln x)^2 + C$

50. Which of the following is $\int x\sqrt{x+1}\, dx$?

(a) $\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

(b) $\frac{1}{2}x^2 \cdot \frac{2}{3}(x+1)^{3/2} + C$

51. Figure 6 contains the graph of a function $F(x)$. On the same coordinate system, draw the graph of the function $G(x)$ having the properties $G(0) = 0$ and $G'(x) = F'(x)$ for each x.

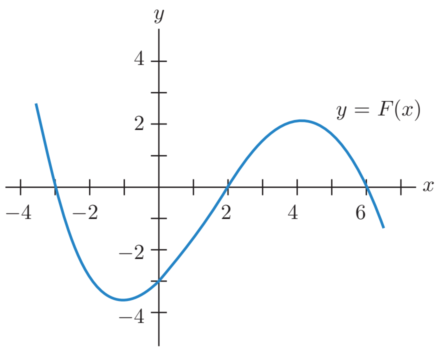

Figure 6

52. Figure 7 contains an antiderivative of the function $f(x)$. Draw the graph of another antiderivative of $f(x)$.

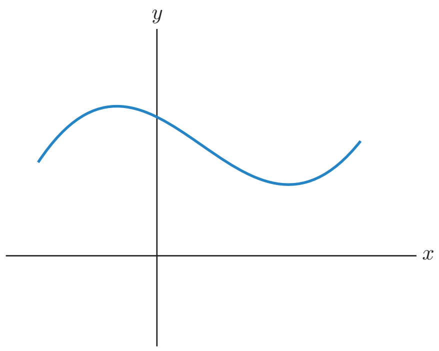

Figure 7

53. The function $g(x)$ in Fig. 8 resulted from shifting the graph of $f(x)$ up 3 units. If $f'(5) = \frac{1}{4}$, what is $g'(5)$?

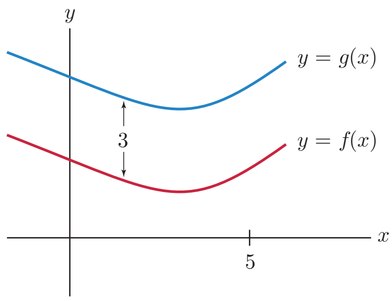

Figure 8

54. The function $g(x)$ in Fig. 9 resulted from shifting the graph of $f(x)$ up 2 units. What is the derivative of $h(x) = g(x) - f(x)$?

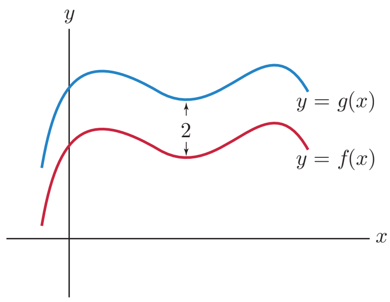

Figure 9

55. Position of a Ball A ball is thrown upward from a height of 256 feet above the ground, with an initial velocity of 96 feet per second. From physics it is known that the velocity at time t is $v(t) = 96 - 32t$ feet per second.

(a) Find $s(t)$, the function giving the height of the ball at time t.

(b) How long will the ball take to reach the ground?

(c) How high will the ball go?

56. Free Fall A rock is dropped from the top of a 400-foot cliff. Its velocity at time t seconds is $v(t) = -32t$ feet per second.

(a) Find $s(t)$, the height of the rock above the ground at time t.

(b) How long will the rock take to reach the ground?

(c) What will be its velocity when it hits the ground?

57. Rate of Production Let $P(t)$ be the total output of a factory assembly line after t hours of work. If the rate of production at time t is $P'(t) = 60 + 2t - \frac{1}{4}t^2$ units per hour, find the formula for $P(t)$.

58. Rate of Production After t hours of operation, a coal mine is producing coal at the rate of $C'(t) = 40 + 2t - \frac{1}{5}t^2$ tons of coal per hour. Find a formula for the total output of the coal mine after t hours of operation.

59. Heat Diffusion A package of frozen strawberries is taken from a freezer at $-5°C$ into a room at $20°C$. At time t, the average temperature of the strawberries is increasing at the rate of $T'(t) = 10e^{-.4t}$ degrees Celsius per hour. Find the temperature of the strawberries at time t.

60. Epidemic A flu epidemic hits a town. Let $P(t)$ be the number of persons sick with the flu at time t, where time is measured in days from the beginning of the epidemic and $P(0) = 100$. After t days, if the flu is spreading at the rate of $P'(t) = 120t - 3t^2$ people per day, find the formula for $P(t)$.

61. Profit A small tie shop finds that at a sales level of x ties per day its marginal profit is $MP(x)$ dollars per tie, where $MP(x) = 1.30 + .06x - .0018x^2$. Also, the shop will lose \$95 per day at a sales level of $x = 0$. Find the profit from operating the shop at a sales level of x ties per day.

62. Cost A soap manufacturer estimates that its marginal cost of producing soap powder is $C'(x) = .2x + 1$ hundred dollars per ton at a production level of x tons per day. Fixed costs are \$200 per day. Find the cost of producing x tons of soap powder per day.

63. U.S. Consumption of Iron Ore The United States has been consuming iron ore at the rate of $R(t)$ million metric tons per year at time t, where $t = 0$ corresponds to 1980 and $R(t) = 94e^{.016t}$. Find a formula for the total U.S. consumption of iron ore from 1980 until time t.

64. U.S. Natural Gas Production Since 1987, the rate of production of natural gas in the United States has been approximately $R(t)$ quadrillion British thermal units per year at time t, with $t = 0$ corresponding to 1987 and $R(t) = 17.04e^{.016t}$. Find a formula for the total U.S. production of natural gas from 1987 until time t.

65. Cost Drilling of an oil well has a fixed cost of \$10,000 and a marginal cost of $C'(x) = 1000 + 50x$ dollars per foot, where x is the depth in feet. Find the expression for $C(x)$, the total cost of drilling x feet. [*Note:* $C(0) = 10{,}000$.]

Technology Exercises

In Exercises 66 and 67, find an antiderivative of $f(x)$, call it $F(x)$, and compare the graphs of $F(x)$ and $f(x)$ in the given window to check that the expression for $F(x)$ is reasonable. [That is, determine whether the two graphs are consistent. When $F(x)$ has a relative extreme point, $f(x)$ should be zero; when $F(x)$ is increasing, $f(x)$ should be positive, and so on.]

66. $f(x) = 2x - e^{-.02x}$, $[-10, 10]$ by $[-20, 100]$

67. $f(x) = e^{2x} + e^{-x} + \frac{1}{2}x^2$, $[-2.4, 1.7]$ by $[-10, 10]$

68. Plot the graph of the solution of the differential equation $y' = e^{-x^2}$, $y(0) = 0$. Observe that the graph approaches the value $\sqrt{\pi}/2 \approx .9$ as x increases.

Solutions to Check Your Understanding 6.1

1. (a) $\displaystyle\int t^{7/2}\,dt = \frac{1}{\frac{9}{2}}t^{9/2} + C = \tfrac{2}{9}t^{9/2} + C$

(b)
$$\int\left(\frac{x^3}{3} + \frac{3}{x^3} + \frac{3}{x}\right)dx$$
$$= \int\left(\frac{1}{3}\cdot x^3 + 3x^{-3} + 3\cdot\frac{1}{x}\right)dx$$
$$= \frac{1}{3}\left(\frac{1}{4}x^4\right) + 3\left(-\frac{1}{2}x^{-2}\right) + 3\ln|x| + C$$
$$= \frac{1}{12}x^4 - \frac{3}{2}x^{-2} + 3\ln|x| + C$$

6.2 The Definite Integral and Net Change of a Function

Suppose that we are given the velocity $v(t)$ of an object moving along a straight line and are asked to compute how far the object has moved as t varies from $t = a$ to $t = b$. If $s(t)$ is the position function, then what we are looking for is the number $s(b) - s(a)$, which represents the *net change* of $s(t)$ as t varies from a to b. Even though we do not know $s(t)$, we do know that it is an antiderivative of $v(t)$. As we will see in this section, any antiderivative of $v(t)$, not just $s(t)$, can be used to compute the net change $s(b) - s(a)$. We start with the following important definition.