Quantum Plasmas

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Abstract
In this entry, the field of quantum plasmas, a generic exotic state of highly ionized matter where quantum effects are relevant, is reviewed; for example, by dense plasmas arising in strong laser irradiation of solid targets, in compact astrophysical objects such as white dwarfs or neutron stars, solid-state plasmas, and ultrasmall electronic devices. The early developments in the subject are described. A concise account of the microscopic and macroscopic approaches to quantum plasmas is given. New advances are also mentioned, with an emphasis on relativistic and exchange effects.

INTRODUCTION

A quantum plasma is a system of charged particles where collective and quantum effects arising from wave–particle duality and quantum statistics are dominant. Although in the past, usually the physics of plasmas was considered as a purely classical field; lately, there has been an increased interest in quantum properties of plasmas. Motivations for this change of point of view came from diverse physical examples from the microscopic (e.g., the ongoing miniaturization involving ultrasmall electronic devices or nanoscopic metal clusters) and macroscopic worlds (e.g., degenerated plasmas in the interior of giant planets and compact astrophysical object like white dwarfs and neutron stars). In addition, intermediate scale systems such as the plasmas arising from the irradiation of solid targets with ultra-intense lasers can degenerate, so that a quantum treatment becomes necessary in some instances. This is the case, for instance, in inertial confinement fusion experiments, where the initial strong compression can produce densities comparable to typical solid state conditions. Also through the simulation of astrophysical problems through laboratory, laser-produced plasmas is now an active research area, where quantum effects tend to be unavoidable.

From the most basic perspective, non-relativistic quantum plasmas are described by a statistical ensemble of N-particle wavefunctions satisfying Schrödinger's equation with self-consistent Coulomb interactions. Equivalently, one can consider the associated N-particle density matrix, which solve von Neumann's equation. The Wigner transform of the density matrix yields the Wigner function, which has the advantage of a more direct classical correspondence with the N-body particle distribution function in phase space. From appropriate averages involving these objects (wavefunction, density matrix, or Wigner function), one can calculate all physical quantities like charge, current, and energy densities, among others. A similar microscopic modeling can be enlarged to take into account, spin and relativistic effects.

In the same way as the classical kinetic theory of plasmas sometimes gives more information than actually needed (also at the cost of a more involved analytical treatment), in the quantum case, it is helpful to develop simplified, macroscopic models. This happens if the details of the quantum statistical ensemble or the corresponding Wigner function are not so decisive. For these reasons, of late, there has been much interest on macroscopic models such as quantum hydrodynamical and/or quantum magnetohydrodynamical models for plasmas, as well as on density functional theories. By means of hydrodynamic modeling, one can access the nonlinear regimes of quantum plasmas in a less difficult manner. For instance, conservation laws, nonlinear wave analysis, the construction of nonlinear structures such as solitonic or periodic solutions and the assessment of quantum turbulence are examples where a fluid treatment is found to be a useful starting point.

In spite of the exciting developments, quantum plasma physics is a traditional topic, with notable achievements starting mostly in the middle of the 20^{th} century. For this reason, a section is dedicated to historical notes in this entry. Finally, we include some notes on very new advances, regarding the role of exchange and relativistic effects in quantum plasmas.

This entry is organized as follows. The next section presents the basic parameters for the characterization of quantum plasmas, followed by a discussion of significant work from the literature on quantum plasmas. Next, the microscopic and macroscopic descriptions of quantum plasmas are briefly outlined, with illustrative examples such as the quantum Zakharov equations derived from quantum hydrodynamics (QHDs) for plasmas, describing the nonlinear interaction of high frequency (Langmuir) and low frequency (ion-sound) modes. Subsequently, some of the new advances are shown, such as the incorporation in the Vlasov equation of exchange effects due to particle indistinguishability and a relativistic N-stream model for quantum plasmas, among others. Finally, conclusions are outlined.

Encyclopedia of Plasma Technology DOI: 10.1081/E-EPLT-120053343

BASIC PROPERTIES OF QUANTUM PLASMAS

Here, we discuss the most fundamental parameters characterizing quantum plasmas,restricted to the simplest case of an electron gas embedded in a fixed homogeneous ionic background. Generalization to multi-species plasmas, including ions, positrons, or holes, is not difficult. In general terms, quantum effects in a collection of charged particles can be of two classes: 1) quantum effects due to the wave character of the particles, appearing, e.g., in the wave function spreading, in the tunnel effect or in the uncertainty principle; and 2) quantum effects due to the intrinsic particle indistinguishable character, manifesting (in the case of fermions like the electrons) in the anti-symmetry of the N-body quantum state under permutation of elements. Class 1 refers to quantum diffraction effects, while class 2 refers to quantum statistical or degeneracy effects.

Degeneracy becomes important when the individual electron wave functions have a significant overlap, so that the total quantum state cannot be expressed as a simple tensor product of isolated electron quantum states. Therefore, the fermionic character of the charge character will be evident for sufficiently dense plasmas, satisfying:

$$\lambda_B > n_0^{-1/3} \tag{1}$$

In Eq. 1,

$$\lambda_B = \hbar/(m v_T) \tag{2}$$

is the de Broglie wavelength, where $\hbar = \hbar/(2\pi)$ is the reduced Planck's constant, m is the electron mass, and V_T is the thermal velocity. For a completely ionized gas, λ_B is a measure of the particle wave functions spreading. In terms of the thermodynamic temperature T, one can define,

$$m v_T^2/2 = \kappa_B T \tag{3}$$

where κ_B is Boltzmann's constant. Moreover, in Eq. 1, $n_0^{-1/3}$ is a measure of the inter-particle distance, valid for fully ionized plasma, where n_0 is the equilibrium number density of the electron gases.

Condition 1 can be rewritten alternatively (including the numerical factor $(3\pi^2)^{2/3}$) as:

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n_0)^{2/3} > \kappa_B T \tag{4}$$

In Eq. 4, the quantity E_F is the Fermi energy, which is the energy of an electron at the Fermi surface. From Pauli's exclusion principle, even for zero thermodynamic temperature it is necessary to occupy higher energetic levels, with the top level defining the Fermi surface. In terms of the Fermi temperature $T_F = E_F / \kappa_B$, one then has the degeneracy condition,

$$\chi = \frac{T_F}{T} > 1 \tag{5}$$

where χ is the degeneracy factor. Hence, dense, low-temperature plasmas tend to display a quantum behavior, while dilute, high-temperature plasmas tend to behave classically. As a consequence, for classical plasmas, the Fermi-Dirac distribution of states can be approximated by a Maxwell–Boltzmann distribution. Examples of degenerate plasmas are provided by the electron gases in metals and semi-metals (neutralized by an ionic lattice), dense semiconductors, thin metallic films excited by short laser pulses, plasmas created in intense laser-solid interactions, and the plasma in the interior of giant planets and compact astrophysical objects such as white dwarfs and neutron stars.

Besides the de Broglie wavelength, another notable characteristic spatial scale in dense quantum plasmas is provided by:

$$\lambda_F = \frac{v_F}{\sqrt{3}\omega_p} \tag{6}$$

which is the Thomas-Fermi length λ_F, where,

$$v_F = (2E_F/m)^{1/2} \tag{7}$$

is the Fermi velocity and,

$$\omega_p = [n_0 e^2/(m\epsilon_0)]^{1/2} \tag{8}$$

is the plasma frequency given in terms of the elementary charge e and the vacuum permittivity ϵ_0. The quantity λ_F plays the role of a screening length in degenerate plasmas, in a similar way as the Debye length $\lambda_D = v_T/\omega_p$ for classical plasmas.

The exclusion principle forbids the collision of particles occupying the same quantum state. Hence, in a degenerate plasma, the electron–electron collision rate ν_{ee}, which is typically the most significant collisional contribution, is lowered by a factor of the order of $(T/T_F)^2$, with the rough result,

$$\frac{\nu_{ee}}{\omega_p} \sim \left(\frac{T}{T_F}\right)^2 \frac{E_F}{\hbar\omega_p} \tag{9}$$

The contribution $\hbar\omega_p/E_F$, playing the role of coupling parameter for degenerate plasmas, is the ratio between the plasmon excitation energy $\hbar\omega_p$ and the Fermi energy E_F (which is a measure of the electrons kinetic energy). Dense environments such as in stellar interiors and some inertial confinement fusion schemes can be safely taken as collisionless, with negligible ν_{ee}/ω_p, thanks to large Fermi temperatures. So, unlike classical plasmas, dense quantum plasmas are more ideal for larger densities. This produces enhanced heat conductivity and electron transport. For instance, for the electron gases in metals at room temperature one has the estimates $n_0 \simeq 10^{29} m^{-3}$, $T \simeq 300K \ll T_F \simeq 10^5 K$ and $\nu_{ee}/\omega_p \simeq 10^{-5}$.

So far, the discussion applies to non-relativistic plasmas only. In the case of extreme densities, even the incoherent motion of the particles will be relativistic, with the Fermi velocity becoming comparable to the light velocity c. Or in terms of the Fermi momentum,

$$p_F = \hbar(3\pi^2 n_0)^{1/3} \tag{10}$$

one has a large relativistic parameter $\zeta = p_F/(mc)$. In this case, the Fermi (kinetic) energy assumes the special-relativistic form,

$$E_F = (p_F^2 c^2 + m^2 c^4)^{1/2} - mc^2 \tag{11}$$

which reduces to Eq. 4 in the non-relativistic limit $\zeta \ll 1$. Conditions 2 and 3 are still valid, provided the Fermi energy is reexpressed according to Eq. 11. For instance, for a number density $n_0 \simeq 10^{36} m^{-3}$ (white dwarf) one has $\zeta = 1.2$, while for $n_0 \simeq 10^{39} m^{-3}$ (neutron star) one has $\zeta = 12.0$.

More detailed accounts on the basic properties of quantum plasmas can be found, for instance, in the study of Haas,[1] Shukla and Eliasson,[2,3] Manfredi,[4] and Haas.[5] General treatises on quantum statistical mechanics are found in books.[6,7] The present review does not cover quantum plasmas under very extreme conditions, allowing for quantum electrodynamic processes such as pair creation, or dense nuclear matter and the quark-gluon plasma.[8] Finally, we remark that the above order-of-magnitude discussion on the relevance of quantum effects should be viewed with care in concrete applications. For instance,[9] it can happen that charged particle systems in ultrasmall semiconductor devices behave in a quantum way (e.g., with significant tunneling through heterojunction barriers) even for small values of the degeneracy parameter. This happens because of the short characteristic length associated with the spatial variations of the doping profile or boundary conditions on a nanosized object.

HISTORICAL ANTECEDENTS

Using a collective variables approach, Bohm and Pines[10] applied a series of canonical transformations in order to express the N-body Hamiltonian of an electron gas in an ionic background as a sum of contributions from independent fields (each associated with a "plasmon" excitation) plus a perturbation. In this way, they derived the linear dispersion relation,

$$1 = \omega_p^2 \sum_i \frac{1}{(\omega - k \cdot v_i)^2 - \frac{\hbar^2 k^4}{4m^2}} \tag{12}$$

with a sum over the electron velocities v_i inside the Fermi sphere, or $v_i = |v_i| \leq v_F$, where v_F is the Fermi velocity. Eq. 1 reduces to:

$$\omega^2 = \omega_p^2 + \frac{3}{5} k^2 v_F^2 + \frac{\hbar^2 k^4}{4m^2} + \ldots \tag{13}$$

in the high frequency limit, which gives the quantum Langmuir wave dispersion relation in a fully degenerate electron gas. In Eq. 13, we recognize the salient features of quantum plasmas, namely the influence of the Fermi statistics in the $\sim k^2$ term and of the quantum diffraction in the $\sim k^4$ term.

It should be noted that historically the first derivation of the dispersion relation (8) was provided by Klimontovich and Silin.[11] They considered the quantum Vlasov equation satisfied by the Wigner function in the presence of a self-consistent electrostatic field. Similarly, Lindhard[12,13] has considered the response of fully degenerate quantum plasmas including quantum recoil, for both longitudinal and transverse high frequency waves, with particular emphasis on the imaginary part of the dielectric function as a function of both frequency and wavenumber. The results apply to the collisionless damping and stopping power in the plasma. Although the longitudinal response is more widely known, in connection with solid state plasma applications,[14] lately the transverse response has found to be interesting, e.g., in the discussion of wake fields and energy loss experiments in electron microscopy.[15]

The pioneering work on quantum plasmas have been strongly influenced by field theoretical methods, which is inline with the consolidation of quantum electrodynamics taking place at the time. In this context, Gell-Mann and Brueckner[16] estimated the correlation energy of a degenerate free electron gas in an immobile homogeneous ionic background, to leading order on the Wigner–Seitz radius.[14] They used renormalization techniques to sum the most divergent terms in the perturbation series for the self-energy of the electron gas. The leading order term is the Fermi energy, a measure of the kinetic energy of the system, while the remaining terms correspond to the exchange correlation energy (discussed later). In addition, Kelly[17] derived the dielectric function for magnetized quantum plasmas, using quantum kinetic theory under a Fermi–Dirac statistics assumption.

For more detailed reviews on the old quantum plasma literature, studies by Pines[18]and Drummonds[19] can be consulted. In addition, works by Melrose[20,21] provide extensive references on the previous developments, in the context of *quantum plasmadynamics*, combining quantum electrodynamics and the kinetic theory of charged particle systems, in a covariant description of relativistic quantum plasmas.

MICROSCOPIC MODELING

Non-relativistic quantum systems in general are described by the N-body density operator,

$$\hat{\rho}^N = \sum_\alpha c_\alpha |\psi_\alpha^N\rangle\langle\psi_\alpha^N| \tag{14}$$

where $|\psi_\alpha^N\rangle$ is the normalized α-th N-body quantum state occurring with a probability c_α in the quantum statistical ensemble, while $\langle\psi_\alpha^N|$ is the corresponding dual. By definition,

$$0 \le c_\alpha \le 1, \ \sum_\alpha c_\alpha = 1 \tag{15}$$

The expectation value of an observable $\hat{O}$ is then,

$$< \hat{O} > = \mathrm{Tr}(\hat{O}\hat{\rho}^N) \tag{16}$$

where the trace Tr is performed using some suitable basis of the Hilbert space.

To keep a close resemblance to classical kinetic theory, it is useful to introduce[22] the Wigner transform of the density matrix, yielding the N-body Wigner function,

$$\begin{aligned} f^N(r_1, p_1, ..., r_N, p_N, t) = \\ \frac{N}{(2\pi\hbar)^N} \int ds_1 ... ds_N \ \exp\left(\frac{-i\sum_{i=1}^N p_i \cdot s_i}{\hbar}\right) \\ \times \rho^N\left(r_1 + \frac{s_1}{2}, ..., r_N + \frac{s_N}{2}, r_1 - \frac{s_1}{2}, ..., r_N - \frac{s_N}{2}, t\right) \end{aligned} \tag{17}$$

depending on the position and momenta r_1, p_1, ..., r_n, p_N of the N particles in the plasma, besides time. In Eq. 17, the density matrix elements are:

$$\begin{aligned} \rho^N\left(r_1 + \frac{s_1}{2}, ..., r_N + \frac{s_N}{2}, r_1 - \frac{s_1}{2}, ..., r_N - \frac{s_N}{2}, t\right) = \\ < r_1 + \frac{s_1}{2}, ..., r_N + \frac{s_N}{2} |\hat{\rho}^N| r_1 - \frac{s_1}{2}, ..., r_N - \frac{s_N}{2} > \end{aligned} \tag{18}$$

expressed in coordinate representation. It should be noted that

$$\begin{aligned} < \hat{O} > = \int dr_1 ... dr_N dp_1 ... dp_N O(r_1, ..., r_N, p_1, ..., p_N, t) \\ \times f^N(r_1, p_1, ..., r_N, p_N, t) \end{aligned} \tag{19}$$

where $O(r_1,..., r_N, p_1,...,p_N, t)$ is the phase-space expression obtained from the observable $\hat{O}$ by means of the Weyl transform.[23] Hence, the Wigner function can be used to calculate expectation values, in a similar way as with a classical probability distribution function in phase space. However, it can happen that $f^N < 0$ in certain regions, preventing it to be considered as a strict probability distribution, in spite of being always a real quantity.

From the Schrödinger equation satisfied by the quantum states, it can be shown that the density operator evolve in time according to the von Neumann equation,

$$i\hbar \frac{\partial \hat{\rho}^N}{\partial t} = [\hat{H}, \hat{\rho}^N] \tag{20}$$

where $\hat{H}$ is the Hamiltonian operator of the system and,

$$[\hat{H}, \hat{\rho}^N] = \hat{H}\hat{\rho}^N - \hat{\rho}^N\hat{H} \tag{21}$$

is the commutator. Similarly, the N-body Wigner function can be shown[24] to satisfy a (somewhat complicated) evolution equation similar to the Liouville equation for the N-body classical probability distribution function.

An useful approach is to focus on partial traces of the full N-body density operator or, equivalently, on reduced Wigner functions. For instance, the 1-body- and 2-body-reduced Wigner functions, respectively, are:

$$f^1(r, p, t) = \int dr_2 ... dr_N dp_2 ... dp_N f^N(r, p, r_2, p_2, ..., r_N, p_N, t) \tag{22}$$

$$\begin{aligned} f^2(r, p, r', p', t) = \int dr_3 ... dr_N dp_3 ... dp_N \\ \times f^N(r, p, r', p', r_3, p_3, ..., r_N, p_N, t) \end{aligned} \tag{23}$$

Such quantities are important, because most physical observables are one- or two-particle operators. It can be shown[24] that the reduced Wigner functions obey a chain of equations, with the equation for the N-body reduced Wigner function being dependent on the (N+1)-body reduced Wigner function. This yields the quantum version of the BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) hierarchy satisfied by the reduced probability distribution functions of classical plasma.[25–28]

In the simplest possible case, namely purely electrostatic and collisionless (so that the 2-body Wigner function splits as a product of 1-body Wigner functions), the quantum plasma is described by the Wigner–Poisson system, which reads,

$$\begin{aligned} \frac{\partial f}{\partial t} + \frac{p}{m} \cdot \nabla f + \frac{ie}{\hbar(2\pi\hbar)^3} \int ds\, dp' \exp\left(\frac{i(p - p') \cdot s}{\hbar}\right) \\ \times \left[\phi\left(r + \frac{s}{2}, t\right) - \phi\left(r - \frac{s}{2}, t\right)\right] f(r, p', t) = 0 \end{aligned} \tag{24}$$

$$\nabla^2 \phi = \frac{e}{\epsilon_0}\left(\int dp\, f(r, p, t) - n_0\right) \tag{25}$$

denoting $f(r, p, t) = f^1(r,p,t)$, where $\phi(r, t)$ is the scalar potential and ϵ_0 is the vacuum permittivity. In the formal classical limit $\hbar \to 0$, the system expressed as Eqs. 24 and 25 reduces to the Vlasov–Poisson system of classical plasma, so that it can be called the quantum Vlasov–Poisson system.

It is apparent that the quantum Vlasov–Poisson system is an integro-differential set of equations, due to the nonlocal interaction term in Eq. 24. Extensions to include magnetic field[29] and spin degrees of freedom,[30] even in the collisionless regime, give in principle a framework for the quantum kinetic theory of non-relativistic plasma. Even in this limited perspective, the resulting theories have not been extensively understood, specially regarding the physical interpretation of the several new terms appearing in the evolution equations. It should be noted that the vast majority of the available literature on quantum kinetic theory for plasmas is restricted to linear wave analysis and/or purely electrostatic problems. In particular, to the best of our knowledge, there is not yet an efficient 3-D numerical code for the full Wigner–Maxwell system, which is the quantum version of the Wigner–Maxwell system (the standard setup for classical plasma including magnetic fields).

The above sketch of quantum kinetic theory for plasmas is not at all complete. For instance, instead of the reduced Wigner function approach, one may introduce the concept of nonequilibrium Green functions, leading to the so-called Keldysh-Kadanoff-Baym equations for fermions, plasmons, and photons.[31–33] This and other approaches are not discussed here.

MACROSCOPIC MODELING

The analytic complexity of the microscopic approach suggests the development of simplified macroscopic models, in order to access the nonlinear and electromagnetic (i.e., non-electrostatic) aspects of quantum plasmas. The same is also valid for classical plasmas, where frequently fluid approaches such as magnetohydrodynamics are followed, in spite of the more detailed information provided by the kinetic approach. In this context, the QHD model for plasmas[34,35] is a popular tool. In its simplest form, it reads:

$$\frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0 \tag{26}$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla P}{mn} - \frac{e}{m}(E + u \times B) + \frac{\hbar^2}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) \tag{27}$$

complemented by Maxwell's equation for the self-consistent electromagnetic field,

$$\nabla \cdot E = \frac{e}{\epsilon_0}(n_0 - n), \quad \nabla \cdot B = 0 \tag{28}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla \times B = -\mu_0 e n u + \frac{1}{c^2}\frac{\partial B}{\partial t} \tag{29}$$

where n, u are, respectively, the number density and velocity fields of the electron fluid, P = P(n) is the scalar pressure, E and B are the electric and magnetic fields, n_0 is a neutralizing ionic density background, and μ_0 is the vacuum permeability. In comparison to the classical fluid model, the most evident difference is in the $\sim \hbar^2$ term in Eq. 27, known as the Bohm potential[36] term. This contribution allows to access classically inaccessible regions (as in tunneling events), as well as is responsible for extra dispersion and wave-function spreading. In addition, quantum statistical effects can be incorporated in the equation of state, adapted to the Fermi–Dirac (or Bose–Einstein) nature of the charge carriers.

In terms of the Wigner formulation, no more than two electrons can be accommodated in the same unitary cell (of the size determined by Planck's constant) in phase space, the factor two coming from spin. Hence, for a fixed spatial volume, there will be the need of occupying states of progressively higher momentum. The net effect is a dispersion of velocities, from which the Fermi pressure follows. The picture holds even at zero thermodynamic temperature, with the Fermi pressure arising just from the necessary anti-symmetry of the N-body wave function. The role of the thermodynamic temperature is associated with thermal excitations, allowing occupation of states of energy larger than the Fermi energy. The choice of the pressure function P = P(n) should be adapted to the specific problem, e.g., propagation of quantum Langmuir or quantum ion-acoustic waves.[1]

As in any fluid theory, in the QHD equations for plasmas a long wavelength condition is assumed, in order to be able to neglect kinetic effects such as wave–particle resonance, the plasma echo, and so on. For the QHD model for degenerate plasmas, a rough condition is $\kappa\lambda_F \ll 1$, where k is the wavenumber and λ_F is the Thomas–Fermi length. This has support from molecular simulation in Yukawa fluids.[37] For comparison, in a classical plasma, a necessary condition for the validity of the fluid modeling is $\kappa\lambda_D \ll 1$, where λ_D is the Debye length. However, manifestly such estimates involve the wavenumber, referring to the propagation of specific linear waves only, without mentioning nonlinear or stationary solutions. Similar analyses[37] indicate that the Bohm potential term tend to be less relevant for strongly degenerate plasma, in comparison with the Fermi pressure.

It should be emphasized that contrary to some claims[38] the appearance of the Bohm term in the fluid momentum does not depend on any assumption on the quantum statistical ensemble. Indeed, although in the work of Manfredi and Haas,[35] for simplicity, an hypothesis of equal amplitude of all quantum states was used in the derivation, implying a vanishing osmotic pressure, the same model can be deduced under general conditions, from the moments of the quantum Vlasov equation and eikonal decomposition of the ensemble wave functions. Refer to the study by Haas[34] for the treatment with magnetic fields and to the study by Gasser et al.[39] for the simpler electrostatic case. For discussions about the validity conditions of the QHD model for plasmas, see the works by Haas,[1,5] Manfredi,[4] and Schmidt et al.,[37] and also by Barkery and Ferry[40] on the applicability of QHDs for describing antidot array devices.

The set of Eqs. 26–29 can be improved by the inclusion of phenomenological exchange-correlation[41] or dissipation[42] terms, besides special relativity[43] and spin dynamics[44] effects. QHD models are popular in many different areas, such as in ultrasmall electronic devices modeling,[9,45] nuclear,[46] or molecular physics.[47] The Bohm potential term appear in the form of a gradient correction to the energy-functional density, known in this context as the von Weisäcker[48] term.

As an illustration of the fluid theory, one can consider the quantum Zakharov system[49,50] describing the nonlinear interaction of Langmuir and ion–sound modes[51] in a quantum degenerate electron–ion plasma, allowing for transverse perturbations. The model comprises a set of hydrodynamic equations for classical, cold ions and electrons with Fermi pressure and Bohm potential terms, coupled to Maxwell equations. Performing a two-time scale analysis, one may find the quantum Zakharov system, reading,

$$i\frac{\partial \mathrm{E}}{\partial t} - \frac{5c^2}{3v_F^2}\nabla \times (\nabla \times \mathrm{E}) + \nabla(\nabla \cdot \mathrm{E}) = \mathrm{NE} + \mathrm{H}\nabla\left[\nabla^2(\nabla \cdot \mathrm{E})\right] \tag{30}$$

$$\frac{\partial^2 \mathrm{N}}{\partial t^2} - \nabla^2 \mathrm{N} - \nabla^2(|\mathrm{E}|^2) + \mathrm{H}\nabla^4 \mathrm{N} = 0 \tag{31}$$

where

$$\mathrm{H} = \frac{m}{\mathrm{M}}\left(\frac{5\hbar\omega_p}{3\mathrm{E_F}}\right)^2 \tag{32}$$

is a parameter proportional to the strength of the quantum diffraction effects. In the derivation, the electron fluid is assumed to follow the equation of state for a completely degenerate quantum gas. The classical Zakharov system[52] is obtained when $\mathrm{H} \to 0$, besides replacing $v_F \to v_T$. The still undefined symbols in Eqs. 30–32 are the slow-varying envelope electric field E, the slow part of the density perturbation, N, and the ion mass M. In spite of the small value of the quantum parameter H in most applications, it should be noted that in *nonlinear* events, the total contribution from the quantum terms can be decisive, such as in the arrest of Langmuir wave collapse. This is due to the extra dispersion arising from $\mathrm{H} \neq 0$, as rigorously proven in.[53] Quantum diffraction produces[50] an effective repulsive potential, preventing collapse. The system 22–23 is a starting point for the analysis of nonlinear solutions like bright or dark solitons, vortical states, and related structures.[50]

SOME NEW ADVANCES

In this section, a few very new outcomes in the field are cited, starting with exchange correlation effects in quantum plasma. The exchange correlation energy is the difference between the total interaction energy and the electrostatic energy obtained under an independent particle (or Hartree) assumption. In the Hartree approximation, the two-particle number density is the product of single-particle number densities, neglecting entanglements of any sort.

The exchange correlation energy comes from two influences: on one hand, the Fermi correlation arising from Pauli's principle, which is decisive for electrons of the same spin; this is the exchange energy. On the other hand, there is the Coulomb correlation, which is the more relevant contribution for electrons of same spin; this is named as correlation energy.

Exchange effects in the framework of kinetic theory for quantum plasmas in the low frequency limit have been presented in the work by Zamanian.[54] The available results now indicate that exchange is more relevant for low-frequency processes, such as quantum ion-acoustic wave propagation, while being safely ignorable in high frequency processes, such as the propagation of quantum Langmuir waves. The challenge is to obtain a detailed comparison of this and other conclusions with those from time-dependent density functional theory. While exact in principle, in practice, density functional theories need a phenomenological parametrization of the exchange correlation energy.[55]

Relativistic quantum plasmas are receiving renewed attention, on the analysis of the quantum relativistic wave kinetics,[56] relativistic spin quantum plasmas,[57] the treatment of streaming instabilities, relativistic spin quantum plasmas,[58] and rigorous special relativistic fluid equations for degenerate plasma.[43]

We also remark on the new experimental techniques to probe the quantum aspects of plasmas, thanks to the strong multi-petawatt laser facilities under development. Sensitive measures of the plasma electron density and on the quantum shift on the dispersion relation are becoming available, using X-ray Thomson scattering techniques in high-energy density plasmas.[59,60] In the same context, the laboratory simulation of astrophysical scenarios is also a trend, where quantum plasma effects are relevant.[61] Finally, the relevance of quantum diffraction on particle trapping,[62] the derivation of macroscopic equations using Grad's moments method,[63] and the role of quantum aspects on streaming instabilities in dense plasma,[64] could equally be well mentioned, among others.

CONCLUSION

In this entry, we briefly reviewed the salient characteristics of quantum plasmas, including remarks on the pioneering works, on the basic models, and on future perspectives in the field. The area of quantum plasmas is a traditional subject, attracting attention over the years, at least because of some most natural questions: when and how will a large system of charged particles with collective behavior (a plasma) exhibit quantum behavior? Lately, the accrued urge of interest around quantum plasmas is facilitated due to the development of new experiments allowing, e.g., the assessment of the Fermi pressure and Bohm potential corrections by X-ray irradiation in forward scattering geometry. Besides, new models have been introduced allowing the progressive incorporation of relevant effects, specially regarding spin dynamics, relativistic phenomena, and exchange-correlation energy. Much attention is being paid to the nonlinear collective behavior of quantum plasmas, through analytical and numerical work, toward the identification of new nonlinear structures such as vortices and solitons in such systems. In particular, nonlinear analysis is necessary in the high intensity electromagnetic field regime, such as in plasmas arising under the action of strong laser–matter interactions, and in extreme astrophysics environments. In addition, the increasing relevance of systems on an intermediate scale, such as metal clusters and thin metal films, or possible connections with promising fields such as spintronics, provides further motivation for the development of the quantum plasma area.

ACKNOWLEDGMENTS

We thank the Brazilian research funding agency CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico) for financial support.

REFERENCES

1. Haas, F. *Quantum Plasmas: An Hydrodynamic Approach*; Springer: New York, 2011.
2. Shukla, P.K.; Eliasson, B. Colloquium: Nonlinear collective interactions in quantum plasmas with degenerate electron fluids. Rev. Mod. Phys. **2011**, *83* (3), 885–906.
3. Shukla, P.K.; Eliasson, B. Nonlinear aspects of quantum plasma physics. Phys. Uspheki. **2010**, *53* (1), 51–76.
4. Manfredi, G. How to model quantum plasmas. Fields Inst. Commun. **2005**, *46*, 263–287.
5. Haas, F. An introduction to quantum plasmas. Braz. J. Phys. **2011**, *41* (4), 349–363.
6. Pathria, R.K.; Beale, P.D. *Statistical Mechanics*, 3rd Ed.; Academic Press: Oxford, 2011.
7. Salinas, S. *Introduction to Statistical Physics*; Springer: New York, 2011.
8. Hakim, R. *Introduction to Relativistic Statistical Mechanics —Classical and Quantum*; World Scientific: Singapore, 2011.
9. Di Ventra, M. *Electrical Transport in Nanoscale Systems*; Cambridge: New York, 2008.
10. Bohm, D.; Pines, D. A collective description of electron interactions: III. Coulomb interactions in a degenerate electron gas. Phys. Rev. **1953**, *92* (3), 609–625.
11. Klimontovich, Y.; Silin, V.P.O. spektrakh sistem vzaimodeistvuyushchikh chastits. Zh. Eksp. Teor. Fiz. **1952**, *23*, 151–160.
12. Lindhard, J. On the properties of a gas of charged particles. Dan. Mat. Fys. Medd. **1954**, *28* (8), 1–57.
13. Pines, D.; Nozières, P. *The Theory of Quantum Liquids*; Benjamin: New York, 1966.
14. Mahan, G.D. *Many Particle Physics*; Plenum: New York, 1981.
15. Cockayne, E.; Levine, Z.H. Wake fields in the electron gas including transverse response. Phys. Rev. B **2006**, *74* (23), 235107–235116.
16. Gell-Mann, M.; Brueckner, K.A. Correlation energy of an electron gas at high density. Phys. Rev. **1957**, *106* (2), 364–368.
17. Kelly, D.C. Dielectric tensor for a quantum plasma. Phys. Rev. **1964**, *134* (3A), A641–A649.
18. Pines, D.I. Quantum plasma physics—Classical and quantum plasmas. J. Nucl. En. C: Plasma Phys. **1961**, *2* (5), 5–17.
19. Drummond, J.E. *Plasma Physics*; McGraw-Hill: New York, 1961.
20. Melrose, D.B. *Quantum Plasmadynamics: Unmagnetized Plasmas*; Springer: New York, 2008.
21. Melrose, D.B. *Quantum Plasmadynamics: Magnetized Plasmas*; Springer: New York, 2013.
22. Wigner, E. On the quantum correction for thermodynamic equilibrium. Phys Rev. **1932**, *40* (5), 749–759.
23. Weyl, H. Quantenmechanik und gruppentheorie. Z. Phys. **1927**, *46* (1–2), 1–46.
24. Balescu, R. *Equilibrium and Non-Equilibrium Statistical Mechanics*; Wiley: New York, 1975.
25. Bogoliubov, N.N. Kinetic equations. J. Exp. Theor. Phys. **1946**, *16* (8), 691–702.
26. Born, M.; Green, H.S. A general kinetic theory of liquids I. The molecular distribution function. Proc. Roy. Soc. A **1946**, *188* (1012), 10–18.
27. Kirkwood, J.G. The statistical mechanical theory of transport processes I. General theory. J. Chem. Phys. **1946**, *14* (3), 180–201.
28. Yvon, J. *La Théorie Statistique des Fluides*; Hermann: Paris, France, 1935.
29. Arnold, A.; Steinrück, H. The "electromagnetic" Wigner equation for an electron with spin. J. Appl. Math. Phys. **1989**, *40* (6), 793–815.
30. Zamanian, J.; Marklund, M.; Brodin, G. Scalar quantum kinetic theory for spin-1/2 particles: Mean field theory. New J. Phys. **2010**, *12* (4), 043019–043046.
31. Keldysh, L.V. Diagram technique for nonequilibrium processes. Zh. Eksp. Teor. Fiz. **1964**, *47*, 1515–1527.
32. Kadanoff, L.; Baym, B. *Quantum Statistical Mechanics*; Benjamin: New York, 1962.
33. Bonitz, M. *Quantum Kinetic Theory*; Teubner, Stuttgart, Germany, 1998.
34. Haas, F. A magnetohydrodynamic model for quantum plasmas. Phys. Plasmas **2005**, *12* (6), 062117–062126.
35. Manfredi, G.; Haas, F. Self-consistent fluid model for a quantum electron gas. Phys. Rev. B **2001**, *64* (7), 075316–075323.
36. Bohm, D.; Hiley, B.J. *The Undivided Universe: An Ontological Interpretation of Quantum Theory*; Routledge: London, 1993.
37. Schmidt, R.; Crowley, B.J.B.; Mithen, J.; Gregori, G. Quantum hydrodynamics of strongly coupled fluids. Phys. Rev. E **2012**, *85* (4), 046408–046411.
38. Khan, S.; Bonitz, M. Quantum hydrodynamics. In *Complex Plasmas: Scientific Challenges and Technological Opportunities*; Bonitz, M., Becker, K., Lopez, J., Thomsen, H., Eds.; Springer: New York, 2014, pp. 103–152.
39. Gasser, I.; Lin, C.; Markowich, P.A. A review of dispersive limits of (non) linear Schrödinger type equations. Taiwanese J. Math. **2000**, *4* (4), 501–529.
40. Barkery, J.R.; Ferry, D.K. On the validity of quantum hydrodynamics for describing antidot array devices. Semicond. Sci. Technol. **1998**, *13* (8), A135–A139.
41. Crouseilles, N.; Hervieux, P.-A.; Manfredi, G. Quantum hydrodynamic model for the nonlinear electron dynamics in thin metal films. Phys. Rev. B **2008**, *78* (15), 155412–155423.

42. Jüngel, A.; Tang, S. Numerical approximation of the viscous quantum hydrodynamic model for semiconductors. Appl. Numer. Math. **2006**, *56* (7), 899–915.
43. McKerr, M.; Haas, F.; Kourakis, I. Relativistic theory for localized electrostatic excitations in degenerate electron-ion plasmas. Phys. Rev. E **2014**, *90* (3), 033112–033122.
44. Marklund, M.; Brodin, G. Dynamics of spin 1/2 quantum plasmas. Phys. Rev. Lett. **2007**, *98* (2), 025001–025005.
45. Markowich, P.; Ringhofer, C.A.; Schmeiser, C.A. *Semiconductor Equations*; Springer: Heidelberg, 1990.
46. Fogaça, D.A.; Navarra, F.S. Solitons in relativistic mean field models. Phys. Lett. B **2006**, *639* (6), 629–634.
47. Chan, G.K.; Cohen, A.J.; Handy, N.C. Thomas-Fermi-Dirac-von Weizsäcker models in finite systems. J. Chem. Phys. **2001**, *114* (2), 631–638.
48. Weizsäcker, C.F. Zur theorie der kernmassen. Z. Phys. **1935**, *96*, 431–458.
49. Garcia, L.G.; Haas, F.; de Oliveira, L.P.; Goedert, J. Modified Zakharov equations for plasmas with a quantum correction. Phys. Plasmas **2005**, *12* (1), 012302–012310.
50. Haas, F.; Shukla, P.K. Quantum and classical dynamics of Langmuir wave packets. Phys. Rev. E **2009**, *79* (6), 066402–066411.
51. Haas, F.; Garcia, L.G.; Goedert, J.; Manfredi, G. Quantum ion-acoustic waves. Phys. Plasmas **2001**, *10* (10), 3858–3866.
52. Zakharov, V.E. Collapse of Langmuir waves. J. Exp. Theor. Phys. **1972**, *35* (5), 908–914.
53. Simpson, G.; Sulem, C.; Sulem, P.L. Arrest of Langmuir wave collapse by quantum effects. Phys. Rev. E **2009**, *80* (5), 056405–056414.
54. Zamanian, J.; Marklund, M.; Brodin, G. Exchange effects in plasmas: The case of low-frequency dynamics. Phys. Rev. E **2013**, *88* (6), 063105–063110.
55. Ghosh, S.K.; Deb, B.M. Densities, density-functionals and electron fluids. Phys. Rep. **1992**, *92* (1), 1–44.
56. Mendonça, J.T. Wave kinetics of relativistic quantum plasmas. Phys. Plasmas. **2011**, *18* (6), 062101–062106.
57. Asenjo, F.A.; Muñoz, V.; Valdivia, J.A.; Mahajan, S.M. A hydrodynamical model for relativistic spin quantum plasmas. Phys. Plasmas **2011**, *18* (1), 012107–012117.
58. Haas, F.; Eliasson, B.; Shukla, P.K. Relativistic Klein-Gordon-Maxwell multistream model for quantum plasmas. Phys. Rev. E **2012**, *85* (5), 056411–056425.
59. Glenzer, S.H.; Landen, O.L.; Neumayer, P.; Lee, R.W.; Widmann, K.; Pollaine, S.W.; Wallace, R.J.; Gregori, G.; Holl, A.; Bornath, T. Observations of plasmons in warm dense matter. Phys. Rev. Lett. **2007**, *98* (6), 065002–065005.
60. Glenzer, S.H.; Redmer, R. X-ray Thomson scattering in high energy density plasmas. Rev. Mod. Phys. **2009**, *81* (4), 1625–1663.
61. Cross, J.E.; Reville, B.; Gregori, G. Scaling of magneto-quantum-radiative hydrodynamic equations: From laser-produced plasmas to astrophysics. Astrophys. J. **2014**, *795* (1), 59–69.
62. Daligault, J. Landau damping and the onset of particle trapping in quantum plasmas. Phys. Plasmas **2014**, *21* (4), 040701–040706.
63. Haas, F.; Zamanian, J.; Marklund, M.; Brodin, G. Fluid moment hierarchy equations derived from gauge invariant quantum kinetic theory. New J. Phys. **2010**, *12* (7), 0730270–073040.
64. Bret, A.; Haas, F. Quantum effects in beam-plasma instabilities. In AIP Conference Proceedings, vol. 1421, *International Topical Conference on Plasma Science–Strongly Coupled Ultra-Cold and Quantum Plasmas*; Shukla, P.K., Mendonça, J.T., Eliasson, E., Resendes, D., Eds.; Melville: New York, 2012; 156–163.