Neutrino Oscillations and Instabilities in Degenerate Relativistic Astrophysical Plasmas

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Abstract

We set up a proposal to extend significantly recent works on neutrino-plasma interaction, allowing the possibility of deep degenerate and relativistic electrons, which are often present in compact stars such as high-density white dwarfs. The methodology involves the covariant hydrodynamic formulation of ultra-dense plasmas. We propose the generalization of previous studies, on the interaction between ion-acoustic waves and the resonant neutrino flavor oscillations in a mixed neutrino beam, admitting of degenerate and relativistic electron populations. The destabilization of the ion acoustic wave has higher growth rates, thanks to the very large densities present in plasmas in these extreme conditions. We take into account applications to white dwarf stars in the process of collapsing producing type II supernovae.

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I. INTRODUCTION

Neutrinos play a decisive role in a variety of fields, such as cosmology, particles and astrophysics. The 2015's Nobel Prize, in particular, awarded to T. Kajita and A. B. McDonald for the experimental confirmation of neutrino flavor oscillations, shows the existence of a neutrino mass, pointing out to the incompleteness of the Standard Model.

In the process of gravitational collapse, the extreme densities obtained in the core enhance the inverse beta decay rates, resulting in strong neutrino winds propagating out from the proto-neutron star. Neutrino beams are believed to be responsible for the plasma heating in the stalled shock in type II supernova explosions [1–3]. In this context, for the supernova SN1987A, it was observed a burst of 10^{58} neutrinos of all flavors radiated in a few seconds [4]. Sources of anisotropic neutrino beams have also been investigated [5]. Moreover, neutrinos play a central role in the lepton era of the early universe [6].

Recently, a model for the neutrino-plasma interaction taking into account neutrino oscillations was proposed [7], assuming for simplicity non-degenerate and non-relativistic electrons. However, in compact stars these restrictions are violated easily, with strongly degenerate and relativistic electrons due to Fermi velocities near the speed of light. The purpose of the present work [8] is to improve significantly the previous treatment, in terms of the hydrodynamic model arising from the perfect relativistic fluid equations [9] modified by the neutrino force. Our aim is to consider the destabilization of ion-acoustic waves driven by neutrino beams, evaluating the associate instability growth rates in extreme scenarios. Previously, neutrino oscillations in non-relativistic plasmas have also been analyzed in [10] and [11], taking into account the collisional damping of ion-acoustic waves. In addition, neutrino-magnetohydrodynamic modes [12, 13] and neutrino modified wave propagation in strongly magnetized plasma [14] have been considered, without the inclusion of flavor oscillations, degeneracy or relativistic effects.

The article is organized as follows. In Sec. II, the basic fluid model for the degenerate relativistic electrons and cold non-relativistic ions is written, coupled to a two-flavor neutrino beam taking into account neutrino oscillations. In Sec. III, the linear dispersion relation for ion-acoustic waves is derived. Assuming a double resonance condition between the ion-acoustic wave, the neutrino beam and the neutrino oscillations, the appropriate instability growth rate is obtained. In Sec. IV, the growth rate is evaluated for parameters compatible with type II supernovae. The time-scale of the instability and the unstable wavelengths are calculated, allowing us to estimate the impact of neutrino oscillations. Section V is reserved to the conclusions.

II. BASIC MODEL

For simplicity, we consider a two-flavor neutrino model. The model is given by a hydrodynamical model for electrons, ions, electron-neutrinos and muon-neutrinos. Denoting $n_{e,i}$ and $\mathbf{u}_{e,i}$ as respectively, the electron (e) and ion (i) proper fluid densities and velocity fields, one has the relativistic electrons continuity and force equations,

$$\frac{\partial(\gamma n_e)}{\partial t} + \nabla\cdot(\gamma n_e \mathbf{u}_e) \;=\; 0\,, \quad \gamma = \left(1 - |\mathbf{u}_e|^2/c^2\right)^{-1/2}\,, \tag{1}$$

$$m_e H\left(\frac{\partial}{\partial t} + \mathbf{u}_e\cdot\nabla\right)(\gamma \mathbf{u}_e) \;=\; -\frac{\gamma}{n_e}\left(\nabla + \frac{\mathbf{u}_e}{c^2}\frac{\partial}{\partial t}\right)P + e\nabla\phi + \sqrt{2}G_F(\mathbf{E}_\nu + \mathbf{u}_e\times\mathbf{B}_\nu)\,, \tag{2}$$

together with the corresponding non-relativistic equations for cold ions,

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i\mathbf{u}_i) = 0\,, \quad m_i\left(\frac{\partial}{\partial t} + \mathbf{u}_i\cdot\nabla\right)\mathbf{u}_i = -Z\,e\,\nabla\phi\,. \tag{3}$$

In Eqs. (1)-(3), $m_{e,i}$ are the electron (charge $-e$) and ion (charge $Z\,e$) masses, c is the speed of light, G_F is the Fermi coupling constant and ϕ is the electrostatic potential. In addition, $\mathbf{E}_\nu, \mathbf{B}_\nu$ are effective neutrino electric and magnetic fields defined by

$$\mathbf{E}_\nu = -\nabla N_e - \frac{1}{c^2}\frac{\partial}{\partial t}(N_e\mathbf{v}_e)\,, \quad \mathbf{B}_\nu = \frac{1}{c^2}\nabla\times(N_e\mathbf{v}_e)\,, \tag{4}$$

in terms of the electron-neutrino fluid density and velocity field $N_e, \mathbf{v}_e$. We also have Poisson's equation

$$\nabla^2\phi = \frac{e}{\varepsilon_0}(\gamma\,n_e - Z\,n_i)\,, \tag{5}$$

where ε_0 is the vacuum permittivity constant. The Fermi weak force couples electrons (leptons) to electron-neutrinos. Finally, to determine the pressure $P = P(n_e)$ for a fully degenerate electron gas we use Chandrasekhar's [15] barotropic equation of state,

$$\frac{P}{n_0 m_e c^2} = \frac{1}{8\zeta_0^3}\left[\zeta(2\zeta^2-3)\sqrt{1+\zeta^2}+3\sinh^{-1}\zeta\right], \tag{6}$$

$$\zeta = \frac{\hbar}{m_e c}(3\pi^2 n_e)^{1/3} = \zeta_0\left(\frac{n_e}{n_0}\right)^{1/3}, \quad \zeta_0 = \frac{p_F}{m_e c}, \tag{7}$$

where n_0 is the equilibrium electron number density, $\hbar$ is the reduced Planck constant and $p_F = \hbar(3\pi^2 n_0)^{1/3}$ is the electrons Fermi momentum, yielding the specific enthalpy $H = \int dP/(m_e c^2 n_e) = \sqrt{1+\zeta^2}$, responsible for the relativistic electron mass increase due to a large Fermi velocity. It is important to notice that the Chandrasekhar equation of state gives a linear dispersion for ion-acoustic waves in the absence of neutrinos which agrees with the result from the relativistic Vlasov equation, in the long wavelength limit [16]. In addition, magnetized plasmas can be treated, simply including the magnetic force on electrons and ions [14]. In the case of electromagnetic waves, also the full set of Maxwell equations would be necessary [12, 13].

To close the system, one needs the equations for the two-flavor oscillations. One has [7, 11]

$$\frac{\partial N_e}{\partial t} + \nabla\cdot(N_e \mathbf{v}_e) = \frac{1}{2}N\,\Omega_0\,P_2\,, \quad \frac{\partial N_\mu}{\partial t} + \nabla\cdot(N_\mu \mathbf{v}_\mu) = -\frac{1}{2}N\,\Omega_0\,P_2\,, \tag{8}$$

where $N_\mu, \mathbf{v}_\mu$ are the muon-neutrino fluid density and velocity field, $N = N_e + N_\mu$ is the total neutrino fluid density and P_2 represents the quantum coherence in the flavor polarization vector $\mathbf{P} = (P_1, P_2, P_3)$. In addition, $\Omega_0 = \omega_0 \sin 2\theta_0$, where $\omega_0 = \Delta m^2 c^4/(2\,\hbar\,\mathcal{E}_0)$ with a squared neutrino mass difference Δm^2. Finally, $\mathcal{E}_0$ is the neutrino spinor's energy in the fundamental state and θ_0 is the neutrino oscillations mixing angle. Notice the conservation law

$$\frac{d}{dt}\int (N_e + N_\mu)\,d^3r = 0\,, \tag{9}$$

following for instance from decaying or periodic boundary conditions.

Representing the (ultra) relativistic electron and muon neutrino momenta by $\mathbf{p}_e = \mathcal{E}_e \mathbf{v}_e/c^2$, $\mathbf{p}_\mu = \mathcal{E}_\mu \mathbf{v}_\mu/c^2$, where $\mathcal{E}_e, \mathcal{E}_\mu$ are the corresponding neutrino beam energies, the neutrino force equations read

$$\frac{\partial \mathbf{p}_e}{\partial t} + \mathbf{v}_e \cdot \nabla \mathbf{p}_e \;=\; \sqrt{2}\, G_F \left(-\nabla(\gamma n_e) - \frac{1}{c^2}\frac{\partial}{\partial t}(\gamma n_e \mathbf{u}_e) + \frac{\mathbf{v}_e}{c^2} \times [\nabla \times (\gamma n_e \mathbf{u}_e)] \right) , \tag{10}$$

$$\frac{\partial \mathbf{p}_\mu}{\partial t} + \mathbf{v}_\mu \cdot \nabla \mathbf{p}_\mu \;=\; 0\,. \tag{11}$$

The neutrino-plasma interaction model was derived from an action principle [17, 18], in the absence of flavor oscillations and for non-relativistic electrons, see also [19] for the treatment of neutrino-modified Langmuir waves.

To conclude, the flavor polarization vector $\mathbf{P} = (P_1, P_2, P_3)$ evolves in a material medium according to

$$\frac{\partial P_1}{\partial t} = -\Omega(n_e)P_2\,, \quad \frac{\partial P_2}{\partial t} = \Omega(n_e)P_1 - \Omega_0 P_3\,, \quad \frac{\partial P_3}{\partial t} = \Omega_0 P_2\,, \tag{12}$$

where $\Omega(n_e) = \omega_0[\cos 2\theta_0 - \sqrt{2}\, G_F\, n_e/(\hbar\omega_0)]$. Equation (12) was derived [20, 21] for neutrinos traveling in a fixed, static background, so that $\Omega(n_e)$ involves the proper electron density n_e.

Overall, there are 20 equations defined by Eqs. (1)-(3), (5), (8) and (10)-(12), for 20 variables namely the quantities $n_{e,i}$, $\mathbf{u}_{e,i}$, ϕ, $N_{e,\mu}$, $\mathbf{v}_{e,\mu}$ and $P_{1,2,3}$. We observe that the electron- and muon-neutrino energies $\mathcal{E}_{e,\mu}$ are functions of the corresponding momenta by means of the usual relativistic energy-momentum relation $\mathcal{E}_\nu = (p_\nu^2 c^2 + m_\nu^2 c^4)^{1/2}$, $\nu = e, \mu$. A neutrino mass m_ν is assumed just for the sake of the calculation (at the end it disappears). Specifically, the procedure is detailed in the Appendix A of Ref. [7].

III. LINEAR WAVES

The system (1)-(3), (5), (8) and (10)-(12) admits the equilibrium

$$\begin{aligned} n_e &= n_0\,, \quad n_i = n_0/Z\,, \quad \mathbf{u}_e = \mathbf{u}_i = 0\,, \quad \phi = 0\,, \\ N_e &= N_{e0}\,, \quad N_\mu = N_{\mu 0}\,, \quad \mathbf{v}_e = \mathbf{v}_\mu = \mathbf{v}_0\,, \\ P_1 &= \frac{\Omega_0}{\Omega_\nu}\,, \quad P_2 = 0\,, \quad P_3 = \frac{\Omega(n_0)}{\Omega_\nu} = \frac{N_{e0} - N_{\mu 0}}{N_0}\,, \end{aligned} \tag{13}$$

where $\Omega_\nu = \sqrt{\Omega^2(n_0) + \Omega_0^2}$ is the neutrino-flavor oscillation frequency. Linearization and taking plane wave perturbations $\sim \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ and following the same procedure detailed in [7], the result is

$$\omega^2 = c_s^2\, k^2 + \frac{\Delta_e\, c^2\, k^2\, \Lambda(\theta)\, (c^2\, k^2 - \omega^2)}{(\omega - \mathbf{k}\cdot\mathbf{v}_0)^2} + \frac{\Delta\, \Omega_0^2\, \omega\, \mathcal{E}_0\, (c^2\, k^2 - \omega\, \mathbf{k}\cdot\mathbf{v}_0)}{2\,\hbar\, \Omega_\nu\, (\omega - \mathbf{k}\cdot\mathbf{v}_0)\, (\omega^2 - \Omega_\nu^2)}\,, \tag{14}$$

where

$$\Delta_e = \frac{2\, G_F^2\, N_{e0}\, n_0}{m_i\, c^2\, \mathcal{E}_0}\,, \quad \Delta = \frac{2\, G_F^2\, N_0\, n_0}{m_i\, c^2\, \mathcal{E}_0}\,, \quad \Lambda(\theta) = \left(1 - \frac{v_0^2}{c^2}\right) \cos^2\theta + \sin^2\theta\,, \tag{15}$$

with $\mathbf{k}\cdot\mathbf{v}_0 = k\, v_0\, \cos\theta$ and where the ion-acoustic speed c_s in fully degenerate relativistic plasmas [16] is given by

$$c_s^2 = \frac{Z p_F^2}{3 m_e m_i \sqrt{1+\xi_0^2}}\,, \tag{16}$$

assuming completely ionized plasma with ionic atomic number Z. In comparison with [7], the only difference is the improved ion-acoustic speed, now adapted to fully degenerate and relativistic plasma, and additionally allowing for $Z \neq 1$. It was assumed $\omega << \omega_{pi} = \sqrt{Z\, n_0\, e^2/(m_i\, \varepsilon_0)}$ and $\mathcal{E}_0 = \mathcal{E}_{e0} \approx \mathcal{E}_{\mu 0}$. By inspection, it is not surprising that the dispersion relation (14) is formally identical to the result in [7] (with a new expression for the ion-acoustic speed), since in the linearization procedure one has $\gamma \approx 1$ due to the absence of relativistic streaming electrons.

Besides the traditional ion-acoustic mode, Eq. (14) shows two contributions, one proportional to Δ_e which is due to the energy seed by the streaming neutrinos, while the term proportional to Δ is due to coupling with the neutrino oscillations. In the absence of squared neutrino mass difference ($\Delta m^2 = 0$) one has $\Omega_0^2 = 0$ so that the last term in Eq. (14) disappears. If we formally set $\Delta = 0$, one regains the result in [22], with a modified ion-acoustic speed, taking into account that $c_s << c$ except for huge densities typical of neutron stars, outside the scope of the model.

IV. INSTABILITY OF ION-ACOUSTIC WAVE

In view of the small value $G_F = 4.62 \times 10^{-62}\,\mathrm{J.\,m^3}$ of the Fermi constant, the neutrino contribution in Eq. (14) will typically be a small perturbation. The neutrino effect on ion-acoustic waves without neutrino oscillations has been essentially performed in [22]. Therefore, it is more convenient to focus on the case where neutrino oscillations can have a significant influence. Hence we assume the double resonance condition

$$\omega \approx c_s\, k = \Omega_\nu = \mathbf{k} \cdot \mathbf{v}_0 \,, \tag{17}$$

enhancing the last term in the right-hand side of the dispersion relation (14). Physically, Eq. (17) shows the (almost) resonance of the ion-acoustic wave with the streaming neutrinos, and with the flavor oscillations, which carry an energy which can be exchanged with the wave.

Assuming Eq. (17) we set

$$\omega = \Omega_\nu + \delta\omega \,, \quad |\delta\omega| \ll \Omega_\nu \,, \tag{18}$$

in Eq. (14). Taking into account ultra-relativistic neutrinos ($v_0 \approx c$), one get $\cos\theta = c_s/c << 1$ (a filamentation-like instability), $\omega << c\,k$, so that $\Lambda(\theta) \approx 1$ and

$$(\delta\omega)^3 = \frac{\Delta_e}{2} \left(\frac{c}{c_s}\right)^4 \Omega_\nu^3 + \frac{\Delta\,\Omega_0^2\,\mathcal{E}_0}{8\hbar} \left(\frac{c}{c_s}\right)^2 \,, \tag{19}$$

where $k = \Omega_\nu/c_s$ was used. The unstable mode corresponds to a growth rate $\Gamma = \mathrm{Im}(\delta\omega) > 0$ given by

$$\Gamma = \left(\Gamma_\nu^3 + \Gamma_{\rm osc}^3\right)^{1/3} \,, \tag{20}$$

where

$$\Gamma_\nu = \frac{\sqrt{3}}{2} \left(\frac{\Delta_e}{2} \left(\frac{c}{c_s}\right)^4\right)^{1/3} |\mathbf{k} \cdot \mathbf{v}_0| \,, \quad \Gamma_{\rm osc} = \frac{\sqrt{3}}{2} \left(\frac{\Delta\,\Omega_0^2\,\mathcal{E}_0}{8\hbar} \left(\frac{c}{c_s}\right)^2\right)^{1/3} . \tag{21}$$

The quantity Γ_ν is associated with the coupling with the streaming neutrinos (with $\Omega_\nu = c_s k$). Flavor oscillations are responsible for $\Gamma_{\rm osc}$, which can have some impact only in the case Eq. (17) is satisfied.

The relative influence of neutrino oscillations is given by

$$\left(\frac{\Gamma_{\rm osc}}{\Gamma_\nu}\right)^3 = \frac{\sqrt{2}}{64} \frac{(\Delta m^2 c^4)^2 \sin^2(2\theta_0)}{(G_F n_0)^3} \left(\frac{c_s}{c}\right)^2 \frac{N_0}{\mathcal{E}_0 N_{e0}} \,. \tag{22}$$

For an initially muonic neutrino beam ($N_{e0} = 0, N_{\mu 0} = N_0$), the flavor oscillations dominate, since in this case at the beginning there are no electron-neutrinos to interact with the plasma. Smaller electron densities and smaller streaming neutrinos energies enhance the flavor oscillations correction. Figure I shows the growth rates for $n_0 = 5 \times 10^{29}\mathrm{cm}^{-3}, N_0 = 10^{36}\mathrm{cm}^{-3}, \mathcal{E}_0 = 1\mathrm{MeV}$ and varying initial electron-neutrino population, to be compared with the time-scale of supernova explosions, around 1 second [3]. It is verified that in denser stars the neutrino oscillations play a less significant role.

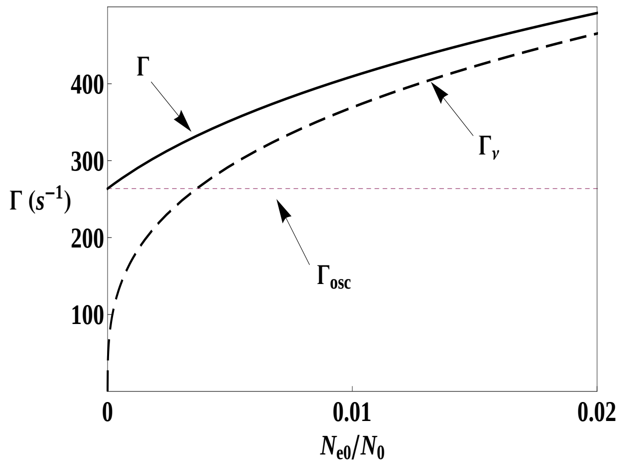

FIG. 1: Continuous line: growth rate Γ from Eq. (19) in terms of the normalized electron-neutrino population. Line-dashed curve: the growth rate Γ_ν which would take place without neutrino oscillations. Horizontal dot-dashed line: the growth rate $\Gamma_{\rm osc}$ due uniquely to neutrino oscillations. Parameters: $n_0 = 5 \times 10^{29}\,\mathrm{cm}^{-3}, N_0 = 10^{36}\,\mathrm{cm}^{-3}, \mathcal{E}_0 = 1\,\mathrm{MeV}$.

The result (22) was derived using the inverted neutrino mass hierarchy assumption

$$\Omega(n_0) \approx -\Omega_\nu \approx -\sqrt{2}\, G_F\, n_0/\hbar\,, \tag{23}$$

which is always satisfied for the dense plasma under consideration. Indeed, taking $\Delta m^2\, c^4 = 3 \times 10^{-5}\,(\mathrm{eV})^2\,, \sin(2\theta_0) = 10^{-1}$ (suitable parameters solving the solar neutrino deficit problem [21]), we find

$$\frac{\sqrt{2} G_F n_0}{\hbar\omega_0} = 2.7 \times 10^{-26}\, n_0\, \mathcal{E}_0 >> 1\,, \tag{24}$$

where n_0 is measured in cm^{-3} and $\mathcal{E}_0$ in MeV, which is the typical energy scale of neutrino beams in supernovae progenitors [4], the inequality following from $n_0 > 10^{28}\mathrm{cm}^{-3}$ in the standard cases [23].

The relativistic effects are depicted in Figure II, for the representative parameters $N_0 = N_{e0} = 10^{35}\,\mathrm{cm}^{-3}, \mathcal{E}_0 = 10\,\mathrm{MeV}$. It is found that the growth rate increases significantly with the plasma density, which also enhances the relativistic effects present in the ion-acoustic speed. For comparison, the results from a non-relativistic fully degenerate model are also shown, setting $\xi_0 \equiv 0$ in Eq. (16). As apparent, the non-relativistic model underestimate the growth-rates by a large amount, specially in denser stars. Notice that in the present case the relativistic character of the plasma arises from the electron degeneracy, so that the parameter ξ_0 can have significant values.

In another potentially relevant limit, the plasma can be relativistic due to a large thermodynamic temperature, but non-degenerate ($T \gg T_F$, where T_F is the Fermi temperature). In this situation, the equation of state would be obtained from the Jüttner distribution, and both the pressure and the number density would involve Bessel functions. Nevertheless, at the end the simple isothermal equation of state $P(n_e) = n_e \kappa_B T$ holds in the non-degenerate case, for arbitrary relativistic strength [24, 25]. Hence the ion-acoustic speed becomes $c_s^{ND} = \sqrt{Z(dP/dn_e)_0/m_i} = \sqrt{Z\kappa_B T/m_i}$. Replacing c_s from Eq. (16) by c_s^{ND}, one has the growth-rate shown in Fig. III, where the parameters are $N_0 = N_{e0} = 10^{35}\,\mathrm{cm}^{-3}, \mathcal{E}_0 = 10\,\mathrm{MeV}$ and $T = 10T_F$ (so that electrons are certainly non-degenerate). There is a qualitative similarity in comparison with the extreme degenerate case (Fig. II). However, the non-degeneracy condition requires large temperatures, shown in Fig. IV.

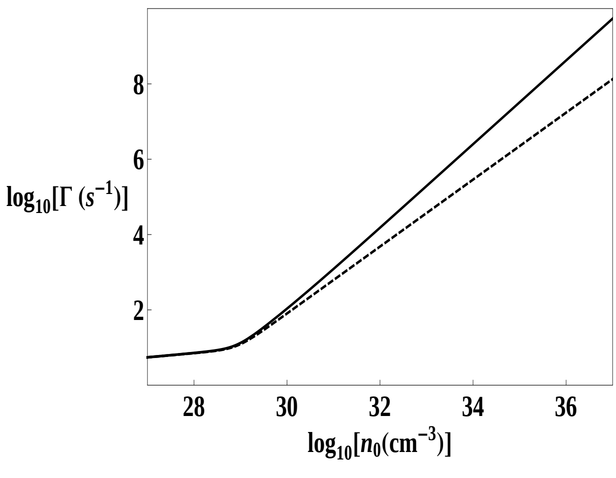

FIG. 2: Continuous line: growth rate Γ from Eq. (19) as a function of plasma density, in logarithmic scale, for iron core stars. Dashed line: the growth rate resulting from the non-relativistic expression of the ion-acoustic speed. Parameters: $N_0 = N_{e0} = 10^{35}\,\mathrm{cm}^{-3}, \mathcal{E}_0 = 10\,\mathrm{MeV}$.

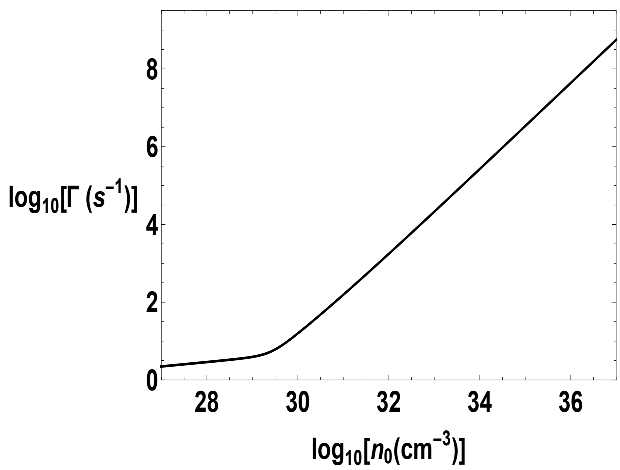

FIG. 3: Growth rate Γ from Eq. (19) in non-degenerate plasma as a function of plasma density, in logarithmic scale, using the non-degenerate ion-acoustic speed $c_s^{ND} = \sqrt{Z\kappa_B T/m_i}$, for iron-core stars. Parameters: $N_0 = N_{e0} = 10^{35}\,\mathrm{cm}^{-3}, \mathcal{E}_0 = 10\,\mathrm{MeV}, T = 10T_F$.

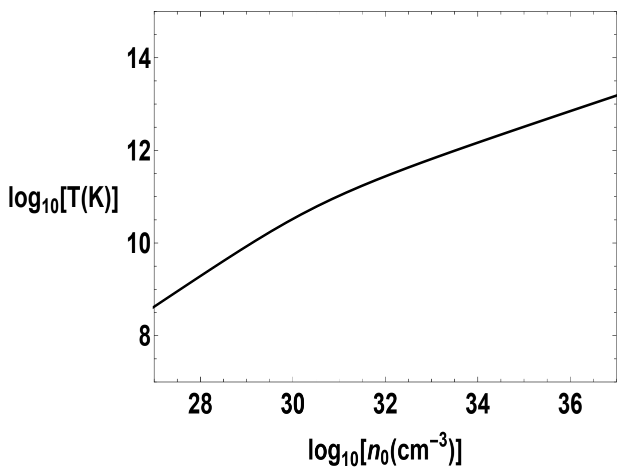

FIG. 4: Thermodynamic temperature $T = 10T_F$, corresponding to Fig. III, as a function of plasma density, in logarithmic scale.

Table I shows the diverse physical quantities for type II core-collapse supernovae electron number densities [4, 5, 23, 26]. For the sake of illustration, we assume an iron core so that henceforth $Z = 26, m_i = 56\,\mathrm{a.m.u.}$. For convenience, the Fermi temperature T_F obtained from $\kappa_B T_F = (p_F^2 c^2 + m_e^2 c^4)^{1/2} - m_e c^2$ is exhibited (noticing the ultra-degeneracy condition $T << T_F$). We observe that the relativistic parameter $\xi_0 = p_F/(m_e c)$ becomes large for very dense systems. The ion-acoustic speed is always seen to be much smaller than the light speed. It can also be verified that $\Omega_\nu >> \omega_{pi}$ as required. In addition, one can define an electron coupling parameter $g = E_p/E_k$, where $E_k \approx \kappa_B T_F$ is a measure of the electrons kinetic energy and $E_p = e^2/(4\pi\epsilon_0 r_s)$ is a measure of the electrons interaction energy in terms of the Wigner-Seitz ratio $r_s = (3/4\pi n_0)^{1/3}$. As apparent from Table I, the electrons ideality condition $g << 1$ is satisfied to a good approximation. Finally, besides the wave frequency $\omega \approx \Omega_\nu$, we also consider the wavelength $\lambda = 2\pi c_s/\Omega_\nu$, to be compared with the typical [27] iron core size ($\sim 30\,\mathrm{km}$). Using the approximation (23), we derive

$$\lambda = \frac{2\pi\hbar c_s}{\sqrt{2}G_F n_0}\,, \tag{25}$$

which always decreases with the density.

$n_0(\mathrm{cm}^{-3})$	$T_F(\mathrm{MeV})$	$p_F/m_e c$	c_s/c	g	$\lambda(\mathrm{m})$
10^{28}	0.02	0.3	2.3×10^{-3}	3.0×10^{-2}	7.2×10^{3}
10^{30}	0.29	1.2	8.8×10^{-3}	8.1×10^{-3}	0.6
10^{32}	2.37	5.5	2.2×10^{-2}	4.5×10^{-3}	0.1
10^{34}	12.68	25.7	4.7×10^{-2}	3.9×10^{-3}	0.03
10^{36}	60.70	119.5	1.0×10^{-1}	3.8×10^{-3}	0.008

TABLE I: Parameters for iron core proto-neutron stars.

Another issue is the possible influence of Landau damping. Evaluating the imaginary part of the longitudinal dielectric function for ion-acoustic waves in ultra-degenerate relativistic plasma, it can be proven [28] that Landau damping is not relevant as long as $p_F/(m_i c) << 1$, which is satisfied except if $n_0 >> 10^{44}\,\mathrm{cm}^{-3}$ (neutron star densities).

V. CONCLUSION

In this work, we reformulate and generalize the treatment of [7], now taking into account the degenerate and relativistic conditions of proto-neutron stars originating type II core-collapse supernovae. The main results are the development of a general model for neutrino-plasma interactions taking into account neutrino flavor oscillations, in degenerate relativistic plasmas. The unstable wavelengths and resonance conditions with neutrino oscillations are found, destabilizing ion-acoustic waves in extreme relativistic astrophysical scenarios. In comparison with the free energy source from the streaming neutrinos, the neutrino oscillations play a more significant role in the ion-acoustic wave destabilization, only if the initial non-electronic neutrino population dominates. No quantum diffraction (Bohm potential) was included because it can be estimated [7] as an exceedingly small contribution for the wavelengths where the double resonance condition (17) is meet. Likewise, exchange-correlation potentials [29, 30] was not yet considered, since to our knowledge, currently there are no known relativistic quantum hydrodynamical equations with exchange-correlation taken into account. Future developments could involve finite electrons temperature, finite neutrino beam temperature and collisional effects. Finally, we expect that the instabilities should enhance the coupling of the neutrino beam with the plasma, specially taking into account the large amplitude waves developing from the unstable linear waves. It would be very interesting to compare the resulting coupling with the ones currently considered in core-collapse supernova models.

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[1] R. Bingham, J. M. Dawson, J. J. Su and H. A. Bethe, Phys. Lett. A **193**, 279 (1994); J.T. Mendonça, R. Bingham, P.K. Shukla, J.M. Dawson and V.N. Tsytovich, Phys. Lett. A **209**, 78 (1995).

[2] L. O. Silva, R. Bingham, J. M. Dawson, J. T. Mendonça and P. K. Shukla, Phys. Rev. Lett. **83**, 2703 (1999).

[3] L. O. Silva and R. Bingham, J. Cosmology and Astroparticle Phys. **05**, 011 (2006).

[4] K. Hirata *et al.*, Phys. Rev. Lett. **58**, 1490 (1987).

[5] J. M. Laming, New Astronomy **4**, 389 (1999).

[6] T. Tajima and K. Shibata, *Plasma Astrophysics* (Addison-Wesley, Reading, 1997).

[7] F. Haas, K. A. Pascoal and J. T. Mendonça, Phys. Rev. E **95**, 013207 (2017).

[8] F. Haas, *Neutrino Oscillations in Degenerate Relativistic Plasmas in Extreme Astrophysical Scenarios*, 9th International Conference on the Frontiers of Plasma Physics and Technology

(FPPT-9), Colombo, Sri Lanka, 8-12 April 2019, `https://www.fpptseries.org`.

[9] S. Weinberg, *Gravitation and Cosmology* (John Wiley and Sons, New York, 1972).

[10] J. T. Mendonça and F. Haas, Phys. Plasmas **20**, 072107 (2013).

[11] F. Haas, K. A. Pascoal and J. T. Mendonça, Phys. Plasmas **24**, 052115 (2017).

[12] F. Haas, K. A. Pascoal and J. T. Mendonça, Phys. Plasmas **23**, 012104 (2016); R. Fitzgerald, *Neutrino Magnetohydrodynamics - the Domains of Plasma Physics and Particle Physics meetin Supernova Explosions.* In: Physics Today (11 January, 2016).

[13] F. Haas and K. A. Pascoal, Phys. Plasmas **24**, 092109 (2017).

[14] F. Haas, K. A. Pascoal and J. T. Mendonça, Phys. Rev. D **96**, 023018 (2017).

[15] S. Chandrasekhar, Mon. Not. R. Astron. Soc. **95**, 207 (1935).

[16] F. Haas, J. Plasma Phys **82**, 705820602 (2016).

[17] L. O. Silva, R. Bingham, J. M. Dawson, J. T. Mendonça and P. K. Shukla, Phys. Plasmas **7**, 2166 (2000).

[18] A. J. Brizard, H. Murayama and J. S. Wurtele, Phys. Rev. E **61**, 4410 (2000).

[19] A. Serbeto, Phys. Lett. A **296**, 217 (2002).

[20] F. Suekane, *Neutrino Oscillations - a Practical Guide to Basics and Applications* (Springer, Tokyo, 2015).

[21] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, Chicago, 1996).

[22] A. Serbeto, P. K. Shukla and L. F. Monteiro, Plasma Phys. Control. Fusion **44**, L43 (2002).

[23] N. Yamamoto, Phys. Rev. D **93**, 065017 (2016).

[24] A. Weiss, W. Hillebrandt, H. C. Thomas and H. Ritter, *Cox and Giuli's Principles of Stellar Structure, Advances in Astronomy and Astrophysics* (Cambridge Scientific Publishers, Cambridge, 2006).

[25] G. Faussurier, Phys. Plasmas **23**, 122704 (2016).

[26] T. Yoshida, K. Takahashi and H. Umeda, J. Phys. Conf. Series **718**, 062073 (2016).

[27] S. E. Wooley and H. -T. Janka, Nature Phys. **1**, 147 (2005).

[28] A. E. Delsante and N. E. Frankel, Ann. Phys. **125**, 137 (1980).

[29] N. Crouseilles, P.-A. Hervieux and G. Manfredi, Phys. Rev. B **78**, 155412 (2008).

[30] F. Haas, Plasma Phys. Control Fusion **61**, 044001 (2019).