On the interaction of a negative-ion beam with ultradense plasma: linear beam-plasma instability and electrostatic soliton characteristics

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Abstract

Ion beam-plasma interaction is an area of fundamental importance in the physics of charged matter. In the ultra-high density and ultra-low temperature limit, quantum effects (manifested via electron degeneracy) become significant and the classical treatment is invalidated. In this paper, we investigate, from first principles, the occurrence of electrostatic excitations in an ultradense (quantum) electron-ion plasma permeated by a tenuous negative-ion beam. The analytical framework relies on a quantum-hydrodynamic model, which incorporates an electron momentum equation that consistently takes into account the equation of state of the Fermi (degenerate) electron gas.

1. Introduction. We present a multifluid model for a plasma consisting of positive ions and degenerate electrons, in addition to a low-density negative-ion streaming component (*the beam*). A linear dispersion relation can be derived for this model, as a basis for stability analysis, revealing the existence of two unstable modes [3]. Based on a relativistic version of the Sagdeev pseudopotential method [1], we have investigated the existence of nonlinear localized solitary excitations in the presence of a negative ion beam. The existence diagram for electrostatic solitary waves (ESWs) will be elaborated, and the structural characteristics (e.g. polarity, amplitude) of electrostatic pulses will be discussed. Bipolar electric field forms are thus obtained, qualitatively reminiscent of earlier experimental observations [2].

2. Theoretical model. We consider a three-component plasma consisting of a dominant ion population (mass m_i, positive charge $q_i = +Z_i e$), a secondary ion species, representing a tenuous beam (mass m_b, charge $q_b = -Z_b e$) and electrons (mass m_e, charge $-e$); e denotes the elementary (absolute) charge, as usual. Spatial variation of the plasma plasma state variables is assumed to occur only in the longitudinal direction, hence the plasma dynamics can be described by a one-dimensional (1D) geometry for simplicity. Our study relies on a multifluid approach, to be introduced in the following paragraph. It is assumed from the outset that magnetic field generation may be neglected within the electrostatic approximation, implying that the total current is negligible (quiescent plasma). Our description follows closely the electrostatic relativistic model proposed in [1, 2, 3], while incorporating the negative-ion beam therein.

The reduced (dimensionless) set of fluid equations reads:

$$\frac{\partial(\gamma_i n_i)}{\partial t} + \frac{\partial}{\partial x}(\gamma_i n_i u_i) = 0, \tag{1}$$

$$\frac{\partial(\gamma_i u_i)}{\partial t} + u_i \frac{\partial(\gamma_i u_i)}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial(\gamma_e n_e)}{\partial t} + \frac{\partial}{\partial x}(\gamma_e n_e u_e) = 0, \tag{3}$$

$$H\left[\frac{\partial(\gamma_e u_e)}{\partial t} + u_e \frac{\partial(\gamma_e n_e)}{\partial x}\right] = \frac{1}{\mu_e}\frac{\partial \phi}{\partial x} - \frac{n_e \gamma_e}{H \mu_e}\left(\frac{\partial n_e}{\partial x} + \alpha u_e \frac{\partial n_e}{\partial t}\right), \tag{4}$$

$$\frac{\partial(\gamma_b n_b)}{\partial t} + \frac{\partial}{\partial x}(\gamma_b n_b u_b) = 0, \tag{5}$$

$$\frac{\partial(\gamma_b u_b)}{\partial t} + u_b \frac{\partial(\gamma_b u_b)}{\partial x} = \frac{1}{\mu_b}\frac{\partial \phi}{\partial x}, \tag{6}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \gamma_e n_e - \beta \gamma_i n_i + \delta \gamma_b n_b \tag{7}$$

where $H = \sqrt{1+\xi^2}$ represents the dimensionless enthalpy of the system [1] (with $\xi = \frac{h n_e}{4 m_e c}$), the relativistic factor is $\gamma_j = 1/\sqrt{1-\alpha u_j^2}$, where $\alpha = c_s^2/c^2 = \mu_e \xi_0^2$, $c_s = \sqrt{2 Z_i E_{Fe}/m_i}$ and $\xi_0 = p_{Fe}/(m_e c) = h n_{e0}/(4 m_e c)$. We have also introduced the ion-to-electron charge ratio $\beta = \frac{Z_i n_{i0}}{n_{e0}} = 1 + \gamma_{b0}\delta$, where γ_{b0} is defined as $\gamma_{b0} = 1/\sqrt{1-\alpha U_{b0}^2}$, the normalized equilibrium beam-fluid speed $U_{b0} = \frac{u_{b0}}{c_s}$, the beam-to-electron charge density ratio $\delta = \frac{Z_b n_{b0}}{n_{e0}}$, the electron-to-ion mass ratio $\mu_e = \frac{m_e}{m_i}$ ($\simeq 1/1836 \ll 1$) and the mass ratio $\mu_b = \frac{m_b}{m_i}$. Overall charge neutrality is assumed at equilibrium (only), imposing $\beta = 1 + \gamma_{b0}\delta$. We shall henceforth typically consider a hydrogen plasma ($Z_i = 1$) and a tenuous beam, i.e. $j_b \approx \gamma_{b0} U_{b0} \delta \ll 1$, with $\mu_b \approx 1$ throughout.

3. Nonlinear analysis. Let us consider a perturbation in the form of a solitary wave propagating with (dimensionless) speed $M = \frac{U_{sol}}{c_s}$, by analogy with the so called *Mach number* in electrostatic soliton theory, i.e. with real (physical) speed U_{sol}. We pass from the laboratory frame to the moving reference frame by assuming that all quantities are functions of a single variable $X = x - Mt$, viz. $\frac{\partial}{\partial t} = -M\frac{\partial}{\partial X}$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial X}$. After some tedious but straightforward algebra, we obtain

$$\frac{1}{2}\left(\frac{d\phi}{dX}\right)^2 + S(\phi) = 0\,. \tag{8}$$

Here, S is a nonlinear function given by

$$S(\phi) = (1+\gamma_{b0}\delta) S_i(\phi) - \delta[S_{b1}(\phi) - S_{b0}] - [S_{e1}(\phi) - S_{e0}]\,, \tag{9}$$

where $S_{e1}(\phi) = \left[\gamma_e n_e\left(\phi + \frac{H_0}{\xi_0^2}\right) - \frac{1}{2\xi_0^3}\left(\sinh^{-1}(\xi_0 n_e) + \xi_0 n_e H\right)\right]$, $S_i(\phi) = M u_i \gamma_i$, $S_{b1}(\phi) = -\mu_b \gamma_{b0} u_b \gamma_b \left(M - U_{b0}\right)$, $S_{b0} = -\mu_b \gamma_{b0}^2 U_{b0}(M - U_{b0})$ and $S_{e0} = \frac{H_0}{\xi_0^2} - \frac{1}{2\xi_0^3}\left(\sinh^{-1}(\xi_0) + \xi_0 H_0\right)$.

4. Existence conditions for electrostatic solitary waves. Pulses are "super-sonic", as imposed by the requirement $S''(\phi = 0; M) \leq 0 \Rightarrow M \geq M_1$; the minimum value (threshold) M_1 is given by

$$(1+\gamma_{b0}\delta)\frac{1}{M_1^2} - \frac{H_0}{1-\mu_e H_0^2 M_1^2} + \frac{\delta}{\mu_b}\frac{1-\alpha U_{b0}\gamma_{b0}^2(M_1 - U_{b0})}{\gamma_{b0}^3(M_1-U_{b0})^2} = 0 \quad . \tag{10}$$

A second physical requirement is associated with the reality of the state variables (density n_j, fluid speed u_j, for $j = e, i, b$), suggesting that $M < M_2$, where the upper bound M_2 is given by the requirement $S(\phi_{max}) \geq 0$, where, for positive and negative pulses, respectively:

$$0 < \phi \leq \phi_{max,i} = \frac{1}{\alpha}\left(1 - \sqrt{1-\alpha M^2}\right)$$

$$\text{and} \qquad 0 > \phi \geq \phi_{max,b} = -\frac{\mu_b}{\alpha}\left[\gamma_{b0}(1-\alpha M U_{b0}) - \sqrt{1-\alpha M^2}\right]. \tag{11}$$

The soliton existence window $[M_1, M_2]$ is investigated in Fig. 1 in terms of the beam parameters.

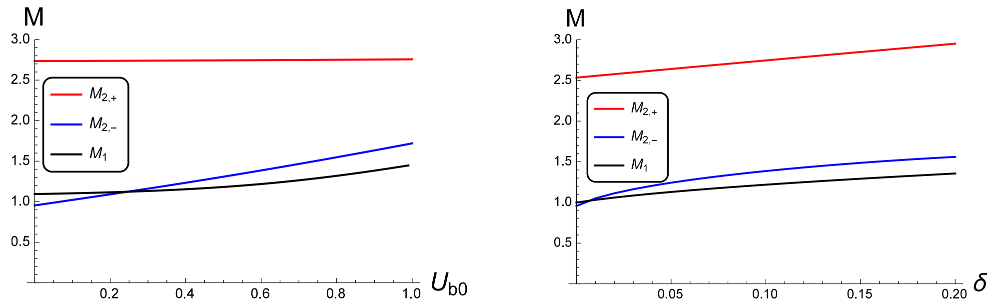
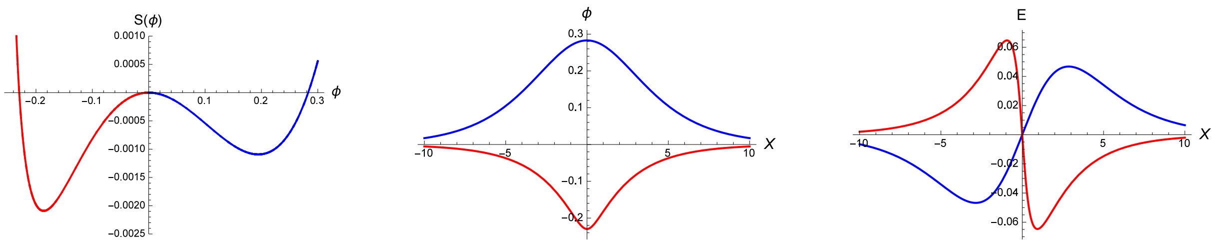


Figure 1: The soliton existence region, is depicted versus U_{b0} (left), with $\delta = 0.1$, and δ (right) with $U_{b0} = 0.6$. Positive pulses occur in the interval $[M_1, M_{2,+}]$; negative pulses in $[M_1, M_{2,-}]$.

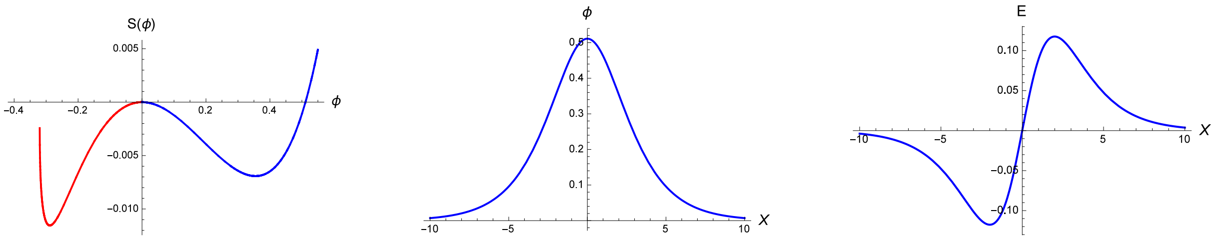


(a) (b) (c)

Figure 2: (a) The pseudopotential $S(\phi)$ is shown in terms of the electrostatic potential ϕ; (b) The electrostatic potential ϕ and (c) the associated electric field E are shown, versus space x. We have taken $n_{e0} = 10^{11} m^{-1}$ (or $\xi_0 \simeq 0.0604$) $U_{b0} = 0.6$, $\mu_b = 1, \delta = 0.1, M = 1.3$.

5. Parametric analysis. Two possibilities exist, as obvious in Fig. 1: either existence of positive pulses (only), or coexistence of positive and negative pulses. We have considered two typical cases for M, below (taking $U_{b0} = 0.6$ as one suitable value, for illustration purposes).

- Case 1: For $M = 1.3$, the pseudopotential $S(\phi)$ is depicted versus ϕ in Fig. 2. The plasma admits coexistence of positive and negative pulses (solitary waves). Two types (polarities) of electrostatic potential ϕ and of the associated electric field are obtained: see Fig. 2.



(a) (b) (c)

Figure 3: (a) The pseudopotential $S(\phi)$ is shown in terms of the electrostatic potential ϕ (note that the negative root has become imaginary, upon increasing the values of M, i.e. the line $M = M_{2,-}$ has been crossed in Fig. 1). (b) The electrostatic potential ϕ and (c) the associated electric field E are shown, versus space x. We have taken $n_{e0} = 10^{11} m^{-1}$ (or $\xi_0 \simeq 0.0604$) $U_{b0} = 0.6$, $\mu_b = 1, \delta = 0.1, M = 1.4$ as indicative values.

- Case 2: For $M = 1.4$, only positive potential pulses occur, as seen in Fig. 3.

6. Conclusion. The nonlinear interaction of a negative ion beam and a infinite homogeneous one-dimensional plasma slab has been investigated from first principles, taking into account quantum (electron degeneracy) and relativistic effects. We have shown that stable supersonic localized pulses (electrostatic solitary waves, ESWs) may form and propagate in the plasma. We have derived explicit conditions for the existence of nonlinear structures. The dynamical effect of the intrinsic beam-plasma (configurational) parameters on the characteristics and on the polarity of ESWs was briefly discussed. Our results extend earlier findings on quantum relativistic plasmas [1, 3, 4].

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