Formulário de cálculo vetorial

1. Produtos triplos

$$\underline{A}\cdot\underline{B}\times\underline{C}=\underline{B}\cdot\underline{C}\times\underline{A}=\underline{C}\cdot\underline{A}\times\underline{B}\,,\qquad \underline{A}\times(\underline{B}\times\underline{C})=\underline{B}\ \underline{A}\cdot\underline{C}-\underline{C}\ \underline{A}\cdot\underline{B}\,.$$

2. Produtos diversos

$$\begin{aligned}\nabla(fg) &= f\nabla g+g\nabla f\,,\qquad \nabla\cdot(f\underline{A})=f\nabla\cdot\underline{A}+\underline{A}\cdot\nabla f\,,\\ \nabla\times(f\underline{A}) &= f\nabla\times\underline{A}+\nabla f\times\underline{A}\,,\qquad \nabla\cdot(\underline{A}\times\underline{B})=\underline{B}\cdot\nabla\times\underline{A}-\underline{A}\cdot\nabla\times\underline{B}\,.\end{aligned}$$

3. Derivadas segundas

$$\nabla\cdot(\nabla\times\underline{A})=0\,,\quad \nabla\times\nabla f=0\,,\quad \nabla\times(\nabla\times\underline{A})=\nabla(\nabla\cdot\underline{A})-\nabla^2\underline{A}\,.$$

4. Teoremas básicos

$$\int_{\underline{a}}^{\underline{b}}\nabla f\cdot d\underline{s}=f(\underline{b})-f(\underline{a})\,,\quad \int\nabla\cdot\underline{A}\,d\tau=\oint\underline{A}\cdot d\underline{a}\,,\quad \int\nabla\times\underline{A}\cdot d\underline{a}=\oint\underline{A}\cdot d\underline{s}\,.$$

5. Coordenadas cartesianas

$$\begin{aligned}d\underline{s} &= dx\,\hat{x}+dy\,\hat{y}+dz\,\hat{z}\,,\quad d\tau=dx\,dy\,dz\,,\quad \nabla f=\frac{\partial f}{\partial x}\hat{x}+\frac{\partial f}{\partial y}\hat{y}+\frac{\partial f}{\partial z}\hat{z}\,,\\ \nabla\cdot\underline{v} &= \frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\,,\quad \nabla\times\underline{v}=\left(\frac{\partial v_z}{\partial y}-\frac{\partial v_y}{\partial z}\right)\hat{x}+\left(\frac{\partial v_x}{\partial z}-\frac{\partial v_z}{\partial x}\right)\hat{y}+\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\hat{z}\,,\\ \nabla^2 f &= \frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}\,.\end{aligned}$$

6. Coordenadas cilíndricas

$$\begin{aligned}x &= =\rho\,\cos\phi,\quad y=\rho\,\mathrm{sen}\phi,\quad z=z\,,\\ \hat{\rho} &= (\cos\phi,\mathrm{sen}\phi,0)\,,\quad \hat{\phi}=(-\mathrm{sen}\phi,\cos\phi,0)\,,\quad \hat{z}=(0,0,1)\,,\\ d\underline{s} &= d\rho\,\hat{\rho}+\rho\,d\phi\,\hat{\phi}+dz\,\hat{z}\,,\quad d\tau=\rho\,d\rho\,d\phi\,dz\,,\quad \nabla f=\frac{\partial f}{\partial\rho}\hat{\rho}+\frac{1}{\rho}\frac{\partial f}{\partial\phi}\hat{\phi}+\frac{\partial f}{\partial z}\hat{z}\,,\\ \nabla\cdot\mathbf{v} &= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho v_\rho)+\frac{1}{\rho}\frac{\partial}{\partial\phi}v_\phi+\frac{\partial}{\partial z}v_z\,,\quad \nabla^2 f=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2}\,,\\ \nabla\times\mathbf{v} &= \left(\frac{1}{\rho}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}\right)\hat{\rho}+\left(\frac{\partial v_\rho}{\partial z}-\frac{\partial v_z}{\partial\rho}\right)\hat{\phi}+\frac{1}{\rho}\left(\frac{\partial(\rho v_\phi)}{\partial\rho}-\frac{\partial v_\rho}{\partial\phi}\right)\hat{z}\,.\end{aligned}$$

7. Coordenadas esféricas

$$
\begin{aligned}
x &= r\,\cos\phi\,\mathrm{sen}\theta\,, \quad y = r\,\mathrm{sen}\phi\,\mathrm{sen}\theta\,, \quad z = r\,\cos\theta\,,\\
\hat{r} &= (\cos\phi\,\mathrm{sen}\theta, \mathrm{sen}\phi\,\mathrm{sen}\theta, \cos\theta), \quad \hat{\theta} = (\cos\phi\cos\theta, \mathrm{sen}\phi\cos\theta, -\mathrm{sen}\theta), \quad \hat{\phi} = (-\mathrm{sen}\phi, \cos\phi, 0),\\
d\underline{s} &= dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\,\mathrm{sen}\theta\,d\phi\,\hat{\phi}\,, \quad d\tau = r^2\mathrm{sen}\theta\,dr\,d\theta\,d\phi\,, \quad \nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r\mathrm{sen}\theta}\frac{\partial f}{\partial\phi}\hat{\phi}\,,\\
\nabla\cdot\underline{v} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\mathrm{sen}\theta}\frac{\partial}{\partial\theta}(\mathrm{sen}\theta\,v_\theta) + \frac{1}{r\mathrm{sen}\theta}\frac{\partial}{\partial\phi}v_\phi\,,\\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\mathrm{sen}\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\mathrm{sen}^2\theta}\frac{\partial^2 f}{\partial\phi^2}\,,\\
\nabla\times\mathbf{v} &= = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\phi\sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial(rv_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(rv_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}\right)\hat{\phi}\,.
\end{aligned}
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