

Formulário de cálculo vetorial

1. Produtos tripos

$$\underline{A} \cdot \underline{B} \times \underline{C} = \underline{B} \cdot \underline{C} \times \underline{A} = \underline{C} \cdot \underline{A} \times \underline{B}, \quad \underline{A} \times (\underline{B} \times \underline{C}) = \underline{B} \underline{A} \cdot \underline{C} - \underline{C} \underline{A} \cdot \underline{B}.$$

2. Produtos diversos

$$\begin{aligned} \nabla(fg) &= f\nabla g + g\nabla f, & \nabla \cdot (f\underline{A}) &= f\nabla \cdot \underline{A} + \underline{A} \cdot \nabla f, \\ \nabla \times (f\underline{A}) &= f\nabla \times \underline{A} + \nabla f \times \underline{A}, & \nabla \cdot (\underline{A} \times \underline{B}) &= \underline{B} \cdot \nabla \times \underline{A} - \underline{A} \cdot \nabla \times \underline{B}. \end{aligned}$$

3. Derivadas segundas

$$\nabla \cdot (\nabla \times \underline{A}) = 0, \quad \nabla \times \nabla f = 0, \quad \nabla \times (\nabla \times \underline{A}) = \nabla(\nabla \cdot \underline{A}) - \nabla^2 \underline{A}.$$

4. Teoremas básicos

$$\int_{\underline{a}}^{\underline{b}} \nabla f \cdot d\underline{s} = f(\underline{b}) - f(\underline{a}), \quad \int \nabla \cdot \underline{A} d\tau = \oint \underline{A} \cdot d\underline{a}, \quad \int \nabla \times \underline{A} \cdot d\underline{a} = \oint \underline{A} \cdot d\underline{s}.$$

5. Coordenadas cartesianas

$$\begin{aligned} d\underline{s} &= dx \hat{x} + dy \hat{y} + dz \hat{z}, & d\tau &= dx dy dz, & \nabla f &= \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}, \\ \nabla \cdot \underline{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}, & \nabla \times \underline{v} &= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z}, \\ \nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}. \end{aligned}$$

6. Coordenadas cilíndricas

$$\begin{aligned} x &= \rho \cos \phi, & y &= \rho \sin \phi, & z &= z, \\ \hat{\rho} &= (\cos \phi, \sin \phi, 0), & \hat{\phi} &= (-\sin \phi, \cos \phi, 0), & \hat{z} &= (0, 0, 1), \\ d\underline{s} &= d\rho \hat{\rho} + \rho d\phi \hat{\phi} + dz \hat{z}, & d\tau &= \rho d\rho d\phi dz, & \nabla f &= \frac{\partial f}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}, \\ \nabla \cdot \underline{v} &= \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho v_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} v_\phi + \frac{\partial}{\partial z} v_z, & \nabla^2 f &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}. \end{aligned}$$

7. Coordenadas esféricas

$$\begin{aligned} x &= r \cos \phi \sin \theta, & y &= \rho \sin \phi \sin \theta, & z &= r \cos \theta, \\ \hat{r} &= (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta), & \hat{\theta} &= (\cos \phi \cos \theta, \sin \phi \cos \theta, -\sin \theta), & \hat{\phi} &= (-\sin \phi, \cos \phi, 0), \\ d\underline{s} &= dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}, & d\tau &= r^2 \sin \theta dr d\theta d\phi, & \nabla f &= \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}, \\ \nabla \cdot \underline{v} &= \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} v_\phi, \\ \nabla^2 f &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}. \end{aligned}$$