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# ANN ESTIMATIONS OF THE ABSORPTION COEFFICIENT IN MULTI-REGION HETEROGENEOUS MEDIA: MOC SOLUTIONS AS TRAINING DATA


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## Introduction

The estimation of the medium absorption coefficient from external measurements can be stated as an inverse problem, and has important applications in optical medicine, including in optical tomography. In this work, we propose a framework based on artificial neural networks (ANNs) to estimate the absorption coefficient in multi-region heterogeneous media. The associated direct transport problem [3] is given as

$$-1<\mu<1, \mu\neq 0: \frac{1}{c}\frac{\partial I}{\partial t}+\mu\frac{\partial I}{\partial x}+\sigma_t I(t,\mu,x)=\sigma_s\Psi(t,x),\ (t,x)\in(0,t_f]\times(a,b), \tag{1a}$$

$$-1<\mu<1: I(0,\mu,x)=0, x\in[a,b], \tag{1b}$$

$$\mu>0: I(t,\mu,x)=q(t,\mu),\ t\in[0,t_f], \tag{1c}$$

$$\mu<0: I(t,\mu,x)=0,\ t\in[0,t_f], \tag{1d}$$

where $I(t,\mu,x)$ $[W/sr]$ is the particle intensity at the time $t$ $[ps]$, in the direction $\mu$ $[sr]$, and at the point $x$ $[cm]$, $c$ $[cm/ps]$ is the average speed of light in the medium, $\sigma_t(x)=\kappa(x)+\sigma_s(x)$ $[1/cm]$ is the total absorption coefficient, $\kappa(x)$ $[1/cm]$ is the absorption coefficient, and $\sigma_s(x)$ $[1/cm]$ is the scattering coefficient. The average scalar flux is denoted by $\Psi(t,x)=\frac{1}{2}\int_{-1}^{1}I(t,\mu',x)\,d\mu'$. The only source is a laser pulse given by [1]

$$q(t,\mu)=w\left(\frac{|\mu-\mu_s|}{\delta_\mu}\right)w\left(\frac{|t-\tau_s-\delta_t|}{\delta_t}\right), \tag{2}$$

where $\mu_s$ is the laser direction, $\delta_\mu$ its angular spread, $\tau_s$ its activation time, $\delta_t$ its temporally center, and $w(\nu)$ is the window function

$$w(\nu)=\begin{cases}1 & ,\nu=0,\\ \exp\left(\left(2e^{-1/|\nu|}\right)/(|\nu|-1)\right) & ,0<\nu<1,\\ 0 & ,|\nu|\geq 1.\end{cases} \tag{3}$$

## Objective

The objective is to estimate the absorption coefficient $\kappa(x)$ from detector measurements $d_0(t)=\Psi(t,a)$ and $d_1(t)=\Psi(t,b)$, $t\in[0,t_f]$.

## Methodology

We propose to estimate $\kappa$ as a piece-wise constant function.

MLP for absorption coefficient estimation

$$\tilde{\boldsymbol{\kappa}}=\mathrm{MLP}(\boldsymbol{d};\boldsymbol{\theta})\approx\boldsymbol{\kappa}=(\kappa_1,\kappa_2,\ldots,\kappa_{n_g}),$$
$$\boldsymbol{d}=(d_0(t_1),d_0(t_2),\ldots,d_0(t_{n_t}),d_1(t_1),d_1(t_2),\ldots,d_1(t_{n_t})),$$

where, $\boldsymbol{\theta}$ are the weights and biases of the multi-layer perceptron (MLP, [2]), $\tilde{\boldsymbol{\kappa}}$ is the estimation of the absorption coefficient, $n_g$ is the number of regions in which $\kappa$ is piece-wise constant, and $n_t$ is the number of time steps in which the detectors measure the scalar flux. The training of the MLP is performed by minimizing the mean squared error (MSE) loss function

$$\mathrm{MSE}(\boldsymbol{\theta})=\frac{1}{n_k n_g}\sum_{s=1}^{n_k}\|\tilde{\boldsymbol{\kappa}}^{(s)}-\boldsymbol{\kappa}^{(s)}\|_2^2,$$

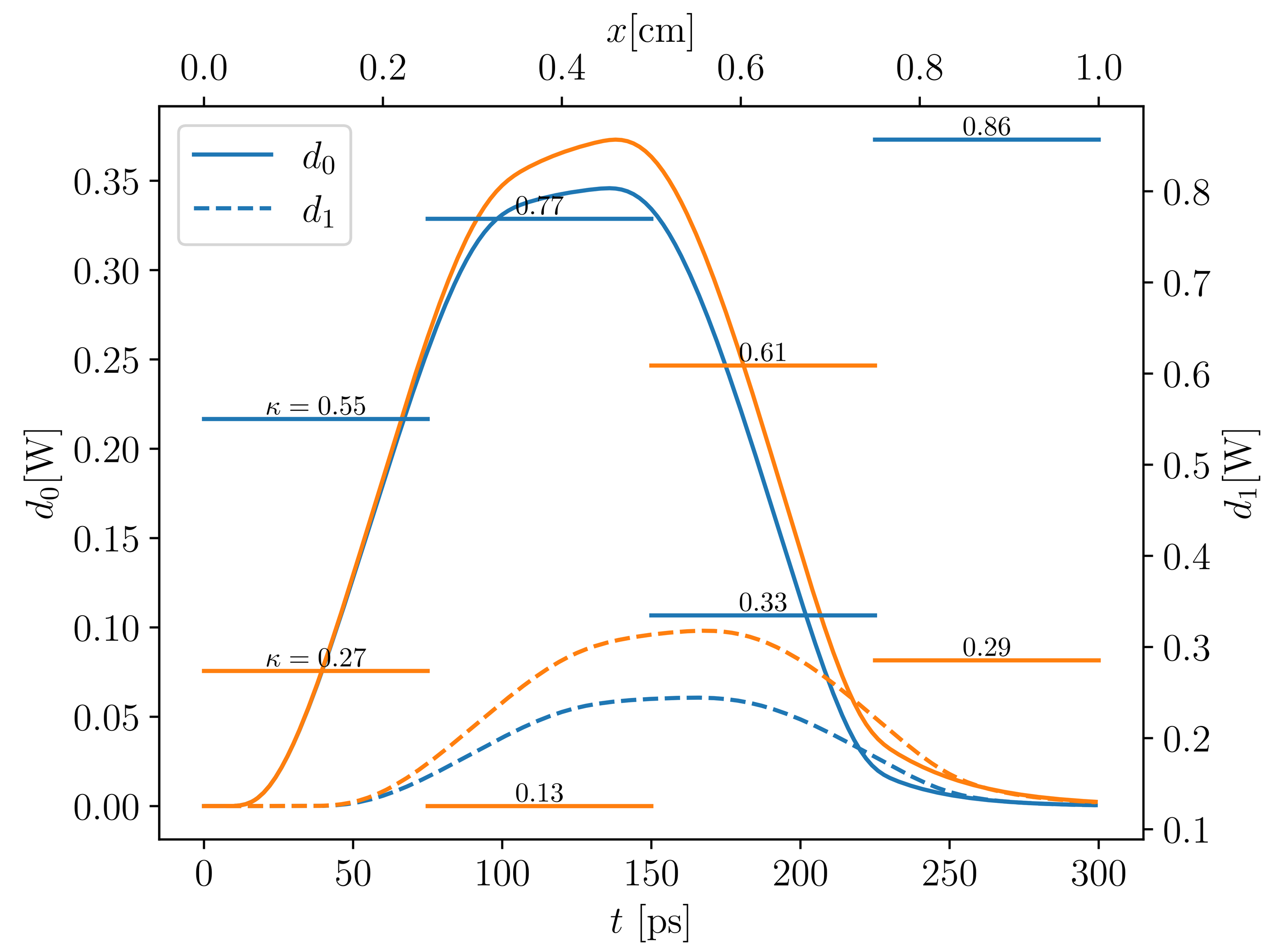
where $n_k$ is the number of samples in the training set, and $\tilde{\boldsymbol{\kappa}}^{(s)}=\mathrm{MLP}(\boldsymbol{d}^{(s)};\boldsymbol{\theta})$ is the estimation of the absorption coefficient for the $s$-th sample. The optimization is performed using the Adam algorithm, which is a stochastic gradient-based method.

Data generation: MoC solutions

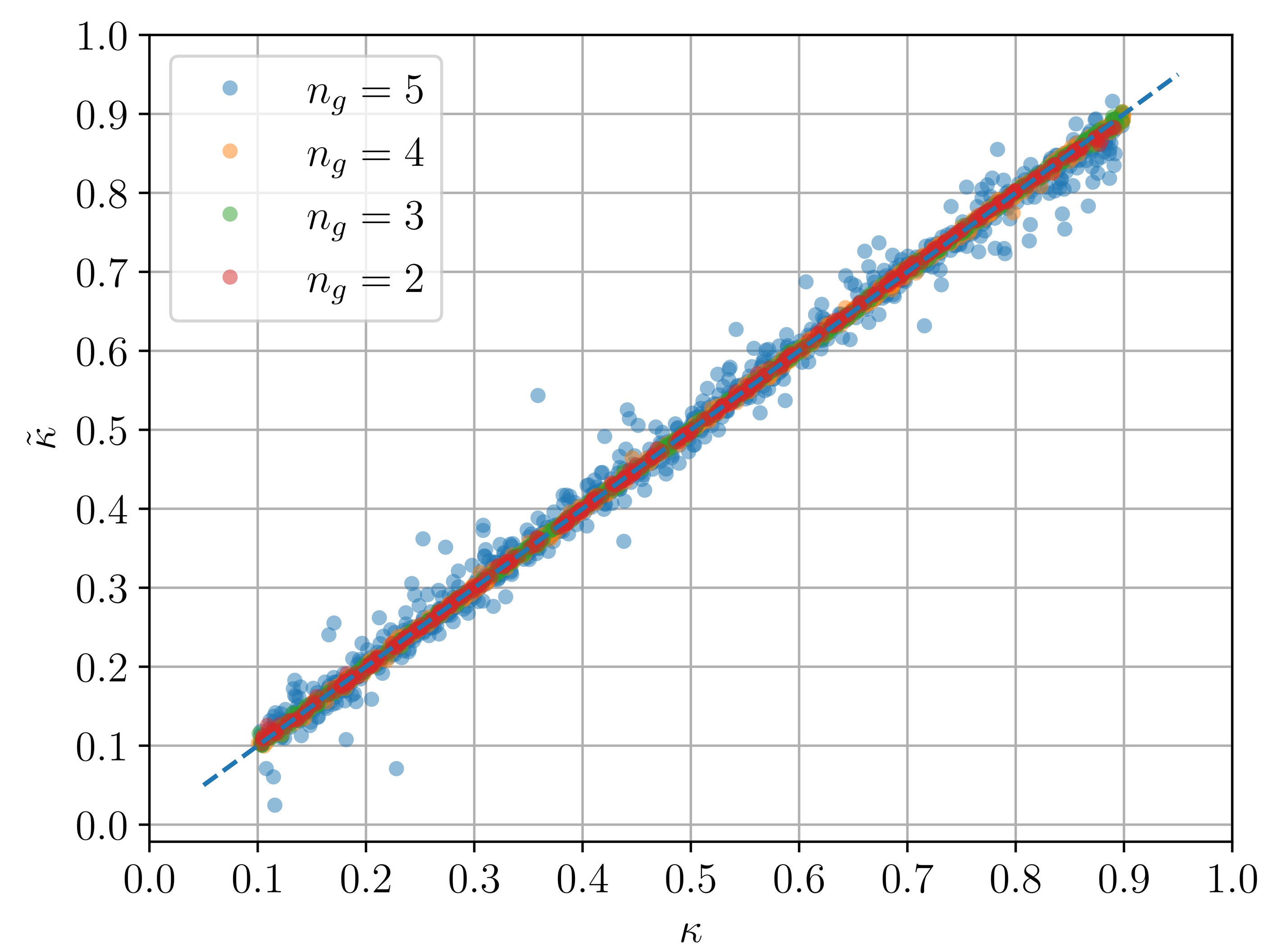
The training and validation sets for the artificial neural network (ANN) of the multilayer perceptron (MLP) type are generated from the solutions of the associate direct problem (1) using the method of characteristics (MoC). The MoC method depends on parameters $n_x$ (number of cells in the computational grid), $N$ (number of Gaussian quadrature pairs), and $h_t$ (time step). Assuming that the properties of the medium are known ($\sigma_s=1$), the sets have been constructed for random values of $0.1\leq\kappa_g<0.9$, distributed in a uniform grid of cells $([x_g,x_{g+1}])_{g=1}^{n_g-1}$, with $n_g\geq 1$ defining the resolution.

## Results

Detectors measurements and the absorption coefficient




Absorption estimations and the grid resolution




## Conclusions

As a work in progress, we have here preliminary test cases. The results indicate that the symmetry assumption of 1D geometry transport is restrictive. Further work should focus in formulating the transport problem in 2D geometry.

## References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