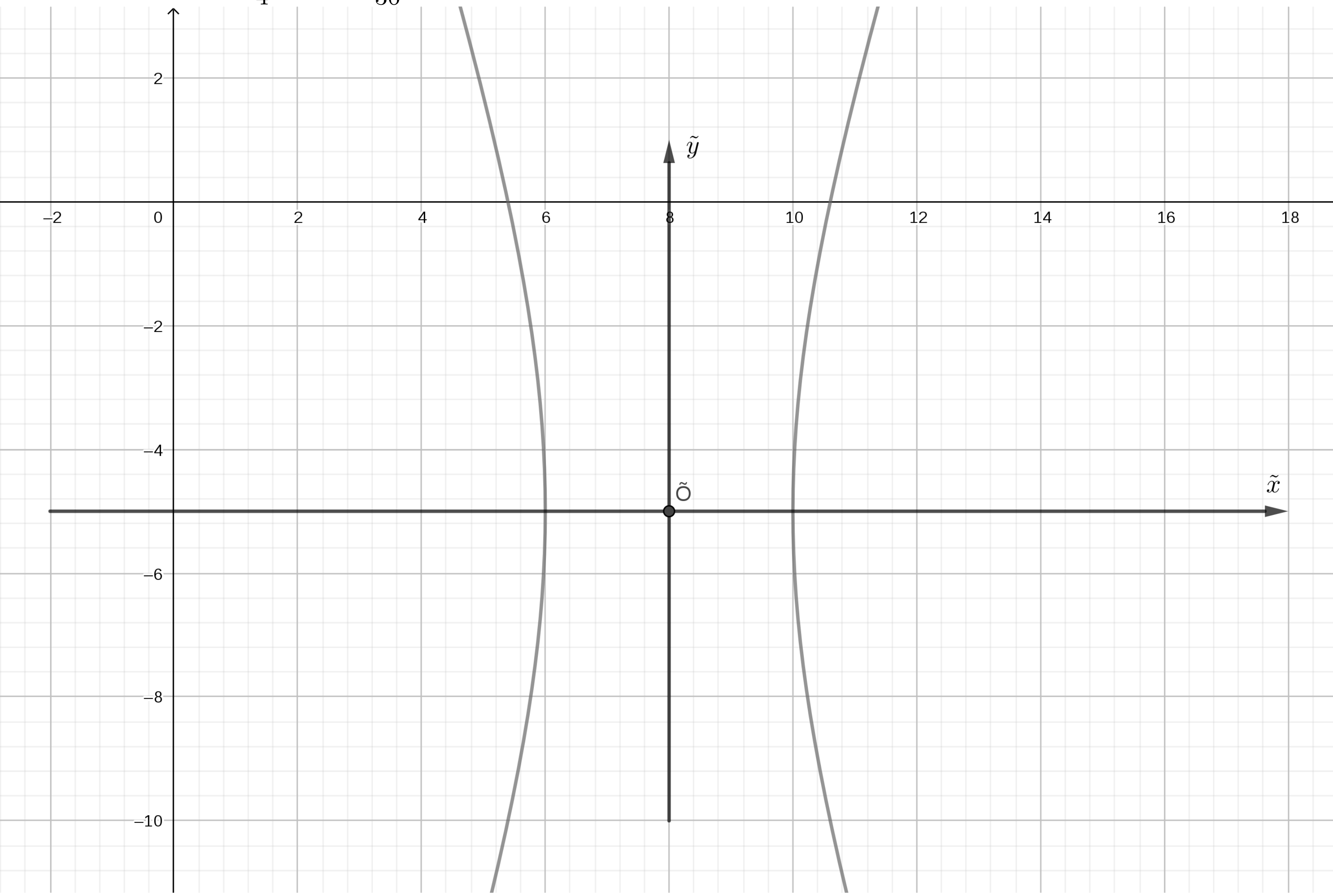
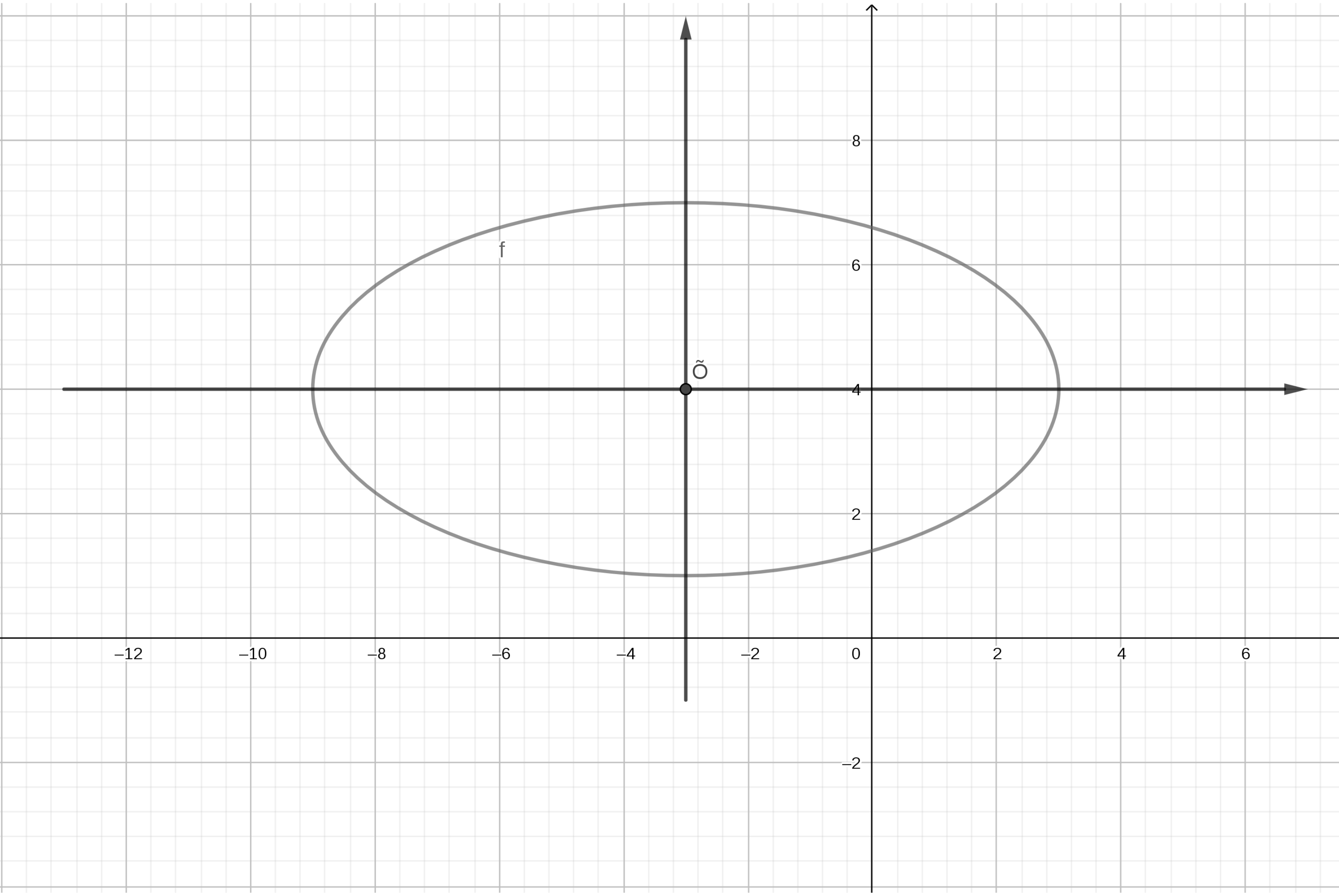
MAT01191 – Vetores e Geometria Analítica – Professora Miriam Telichevesky
Lista de Exercícios 12 – Gabarito

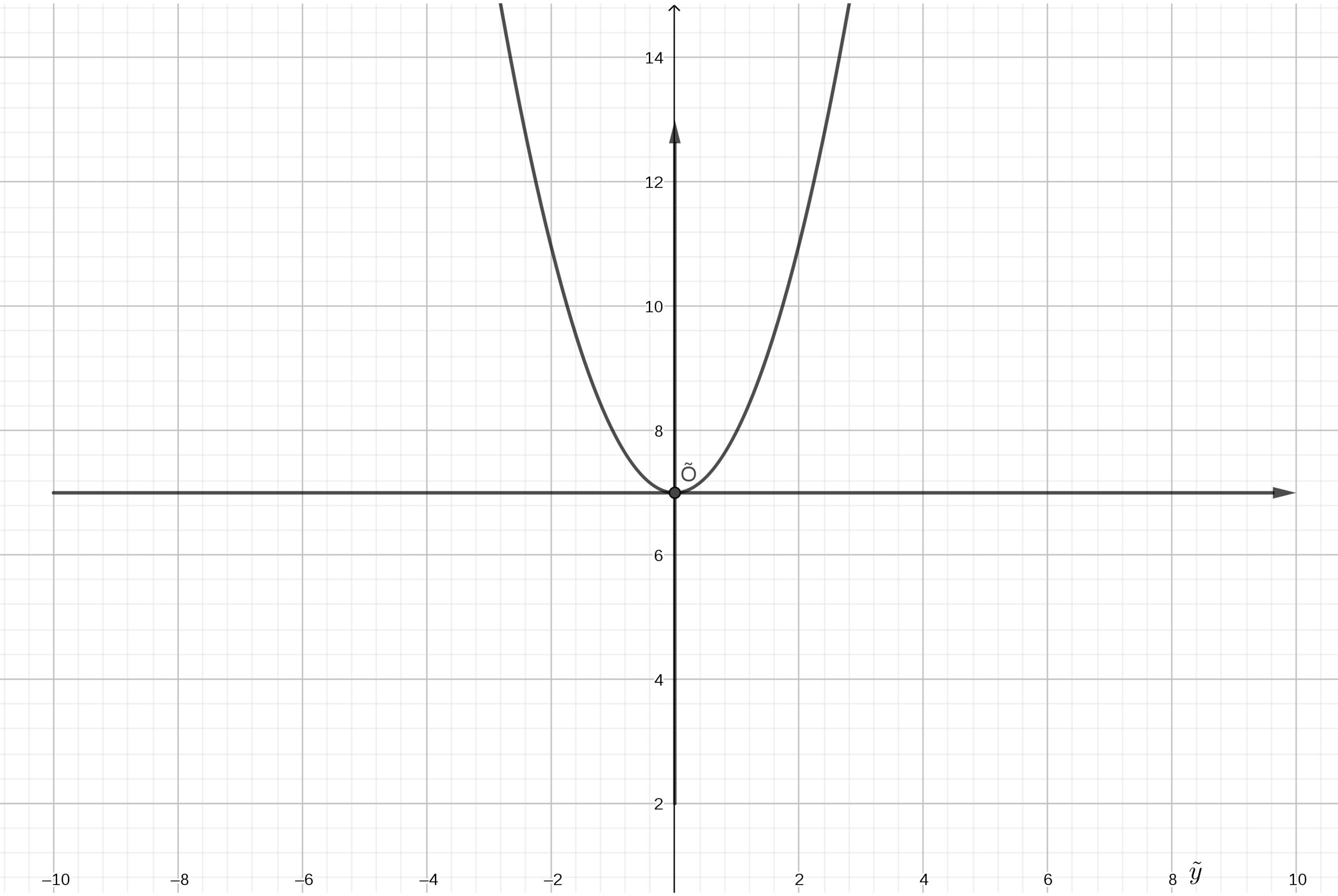
1. (a) $\dfrac{(x-8)^2}{4} - \dfrac{(y+5)^2}{36} = 1$



(b) $\dfrac{(x+3)^2}{36} + \dfrac{(y-4)^2}{9} = 1.$



(c) $x^2 = y - 7$.



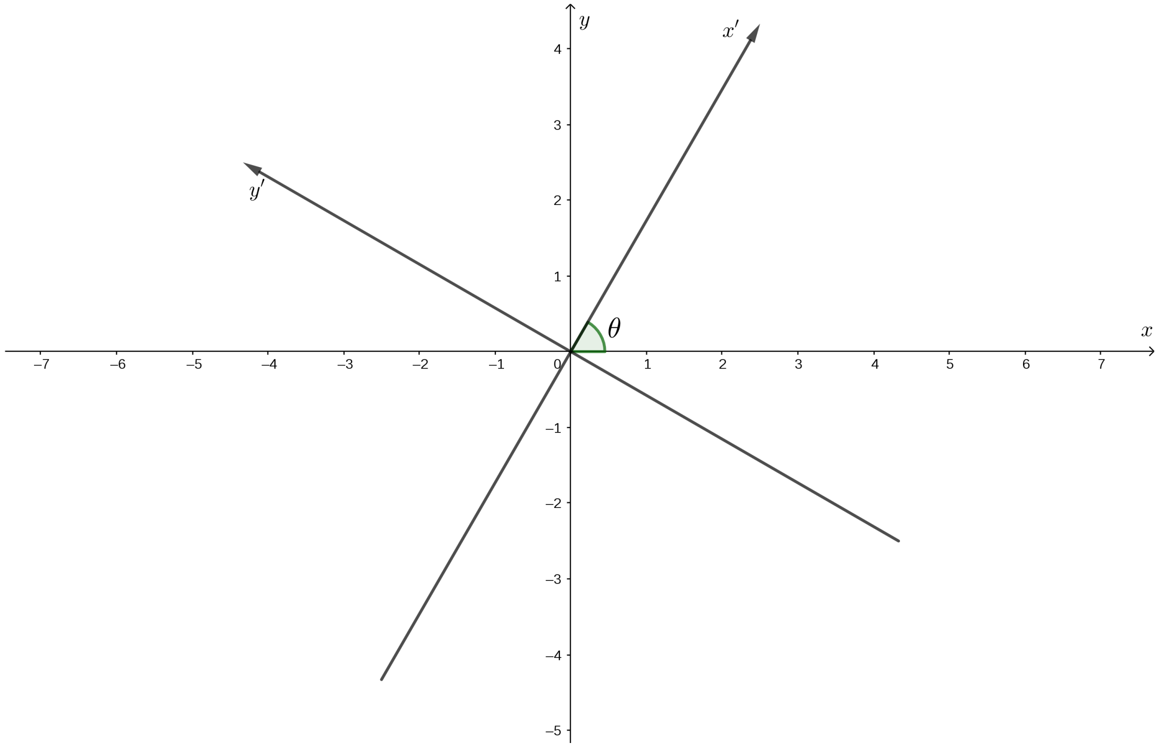
2. (a) $\tilde{x} = x - 4, \tilde{y} = y + 3/2$, a cônica tem equação $\frac{\tilde{x}^2}{20} - \frac{\tilde{y}^2}{25} = 1$.

   (b) $\tilde{x} = x - 3, \tilde{y} = y - 1$, a cônica tem equação $\tilde{x}^2 = 5\tilde{y}$.

   (c) $\tilde{x} = x - 2$, $\tilde{y} = y - 1$, e a cônica tem equação $\tilde{x}^2 + 2\tilde{y}^2 = 7$.

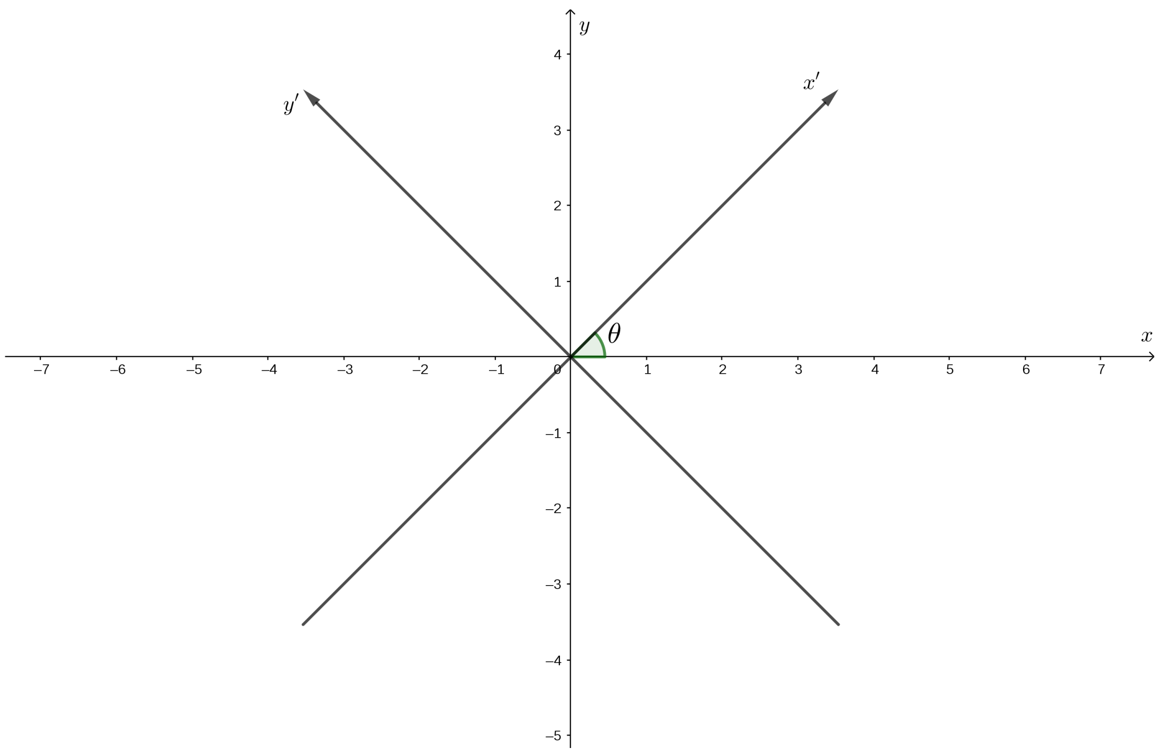
3. (a)

$$\begin{cases} x' = \frac{1}{2}x + \frac{\sqrt{3}}{2}y \\ y' = -\frac{\sqrt{3}}{2}x + \frac{1}{2}y \end{cases}$$

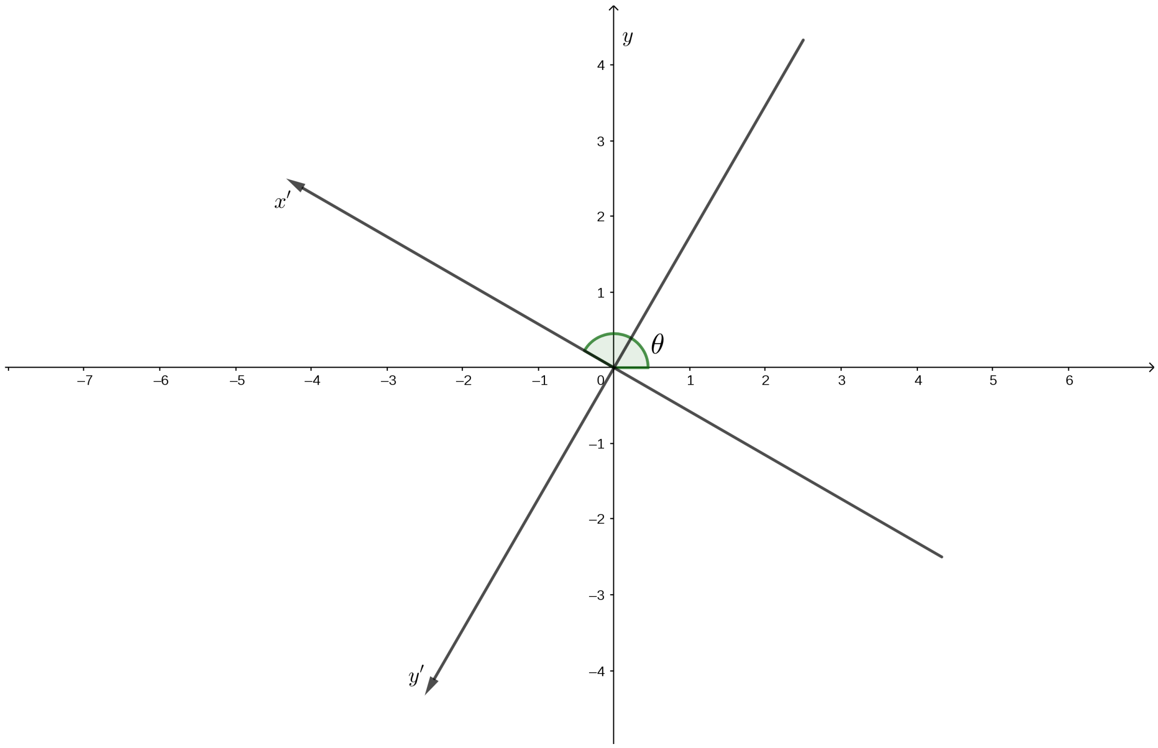


(b)

$$\begin{cases} x' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \\ y' = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \end{cases}$$

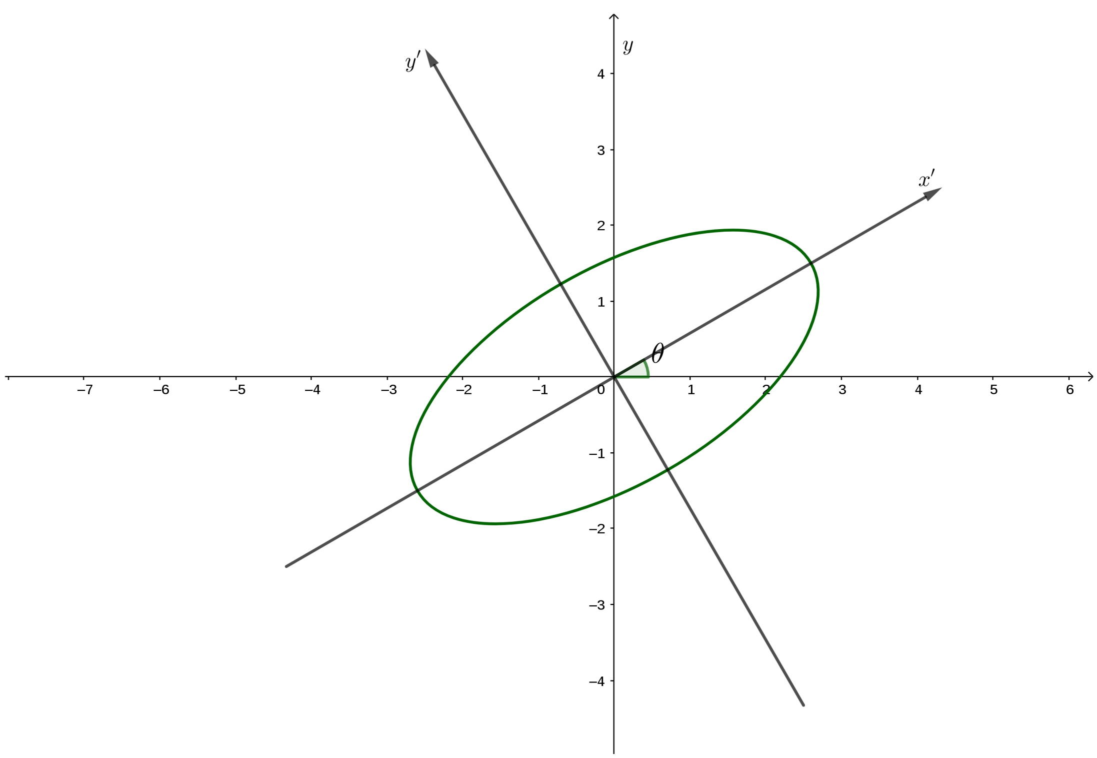


$$\begin{cases} x' = -\frac{\sqrt{3}}{2}x + \frac{1}{2}y \\ y' = -\frac{1}{2}x - \frac{\sqrt{3}}{2}y \end{cases}$$

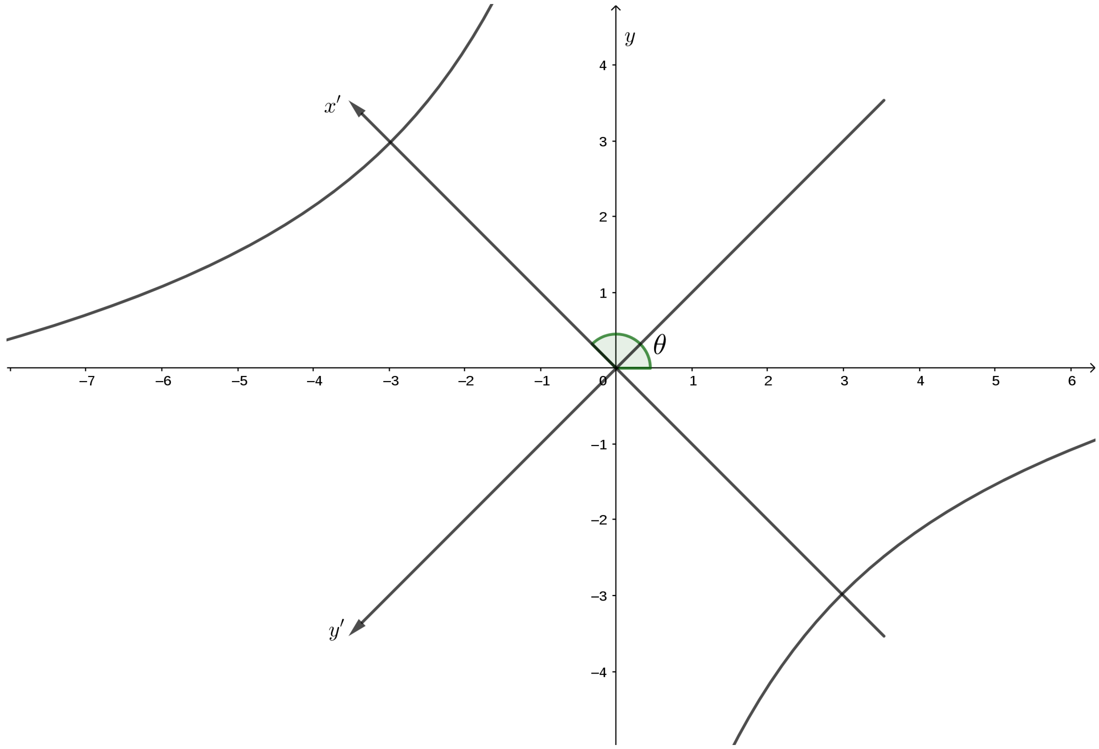


4. (a) $2(3x^2 + 2\sqrt{3}xy + y^2) + 9(x^2 - 2\sqrt{3}xy + 3y^2) - 72 = 0 \iff 15x^2 - 14\sqrt{3}xy + 29y^2 - 72 = 0.$
   (b) $25(x^2 - 2xy + y^2) - 16(x^2 + 2xy + y^2) - 800 = 0 \iff 9x^2 - 72xy + 9y^2 - 800 = 0.$
   (c) ERRATA DA QUESTÃO: DEVERIA SER $x'^2 = -16y'$, $\theta = \pi/2$.
   Resposta: $y^2 = 16x$

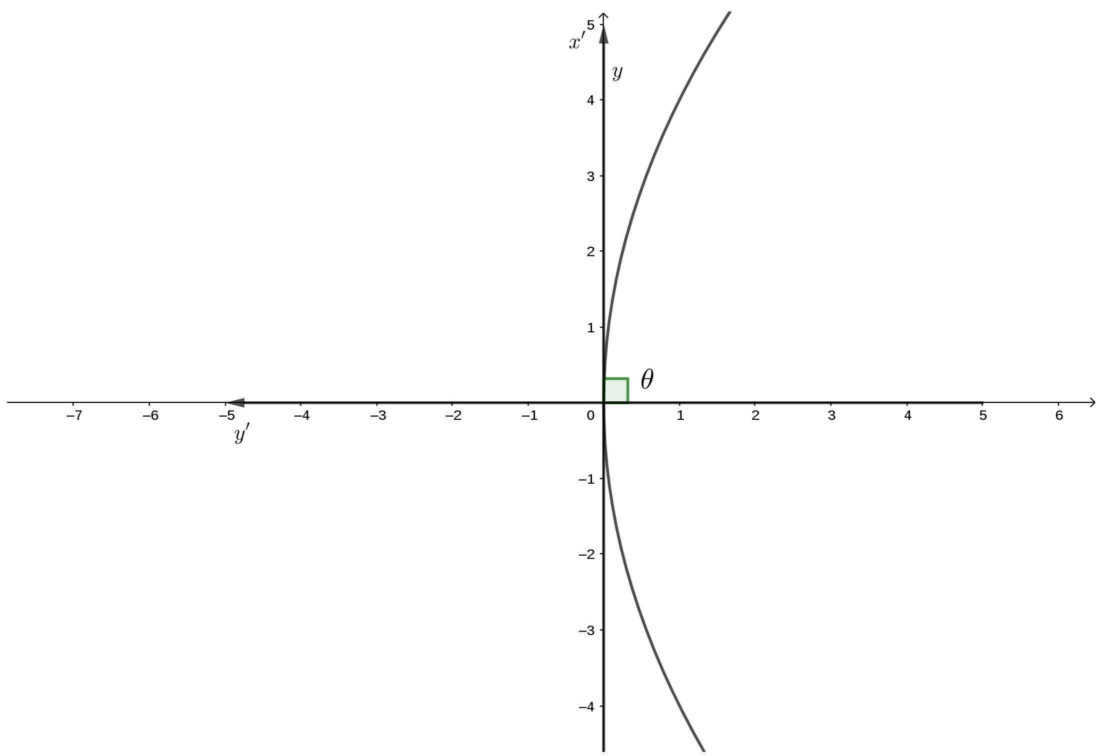
5. (a)



   (b)



(c)



6.

7. IMPORTANTE!!! Ao obter $\cos(2\theta_1)$ e $\tan(2\theta_1)$ é possível descobrir seu quadrante. Para que tenhamos certeza que θ_1 é o menor ângulo de rotação possível é preciso que $2\theta_1$ esteja no I ou II quadrante, ou seja, cos e tan devem ter o mesmo sinal. Isso é impossível de decidir no item (b) pois cos se anula e tangente não existe. Neste caso, vale que $\theta_1 = 45^o$.

   (a) i. $A' = 28$ e $C' = -8$.

ii. $\tan 2\theta_1 = -\sqrt{3}$ e $\cos 2\theta_1 = -\frac{1}{2}$.

iii. $\cos\theta_1 = \frac{1}{2}$ e $\operatorname{sen}\theta_1 = \frac{\sqrt{3}}{2}$.

iv.

$$\begin{cases} x = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' \\ y = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' \end{cases}$$

v. $\dfrac{x'^2}{2} - \dfrac{y'^2}{7} = 1$.

(b) i. $A' = 4$ e $C' = 6$

ii. $\tan 2\theta_1$ não existe e $\cos 2\theta_1 = 0$.

iii. $\cos\theta_1 = \frac{\sqrt{2}}{2}$ e $\operatorname{sen}\theta_1 = \frac{\sqrt{2}}{2}$.

iv.

$$\begin{cases} x = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' \\ y = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' \end{cases}$$

v. $\dfrac{x'^2}{3} + \dfrac{y'^2}{2} = 1$.

(c) i. $A' = 5$ e $C' = -5$.

ii. $\tan 2\theta_1 = \frac{4}{3}$ e $\cos 2\theta_1 = \frac{3}{5}$

iii. $\cos\theta_1 = \frac{2}{\sqrt{5}}$ e $\operatorname{sen}\theta_1 = \frac{1}{\sqrt{5}}$.

iv.

$$\begin{cases} x = \frac{2}{\sqrt{5}}x' - \frac{1}{\sqrt{5}}y' \\ y = \frac{1}{\sqrt{5}}x' + \frac{2}{\sqrt{5}}y' \end{cases}$$

v. $x'^2 - y'^2 = 1$.

(d) i.

ii. $A' = 169$ e $C' = 0$.

iii. $\tan 2\theta_1 = -120/119$ e $\cos 2\theta_1 = -119/169$.

iv. $\cos\theta_1 = \frac{5}{13}$ e $\operatorname{sen}\theta_1 = \frac{12}{13}$.

v.

$$\begin{cases} x = \frac{5}{13}x' - \frac{12}{13}y' \\ y = \frac{12}{13}x' + \frac{5}{13}y' \end{cases}$$

vi. $x'^2 = \dfrac{y'}{13}$.

8. (a) i. $A' = -8$ e $C' = 28$.

ii. $\tan 2\theta_2 = -\sqrt{3}$ e $\cos 2\theta_2 = \frac{1}{2}$.

iii. $\cos\theta_2 = -\frac{\sqrt{3}}{2}$ e $\operatorname{sen}\theta_2 = \frac{1}{2}$.

iv.

$$\begin{cases} x = -\frac{\sqrt{3}}{2}x' - \frac{1}{2}y' \\ y = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' \end{cases}$$

v. $\dfrac{y'^2}{2} - \dfrac{x'^2}{7} = 1$.

(b) i. $A' = 6$ e $C' = 4$

ii. $\tan 2\theta_2$ não existe e $\cos 2\theta_2 = 0$.

iii. $\cos\theta_2 = -\frac{\sqrt{2}}{2}$ e $\operatorname{sen}\theta_2 = \frac{\sqrt{2}}{2}$.

iv.

$$\begin{cases} x = -\frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' \\ y = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' \end{cases}$$

v. $\dfrac{y'^2}{3} + \dfrac{x'^2}{2} = 1$.

(c) i. $A' = -5$ e $C' = 5$.

ii. $\tan 2\theta_2 = \frac{4}{3}$ e $\cos 2\theta_2 = -\frac{3}{5}$

iii. $\cos\theta_1 = -\frac{1}{\sqrt{5}}$ e $\operatorname{sen}\theta_2 = \frac{2}{\sqrt{5}}$.

iv.

$$\begin{cases} x = -\frac{1}{\sqrt{5}}x' - \frac{2}{\sqrt{5}}y' \\ y = \frac{2}{\sqrt{5}}x' - \frac{1}{\sqrt{5}}y' \end{cases}$$

v. $y'^2 - x'^2 = 1$.

(d) i. $A' = 0$ e $C' = 169$.

ii. $\tan 2\theta_2 = -120/119$ e $\cos 2\theta_2 = 119/169$.

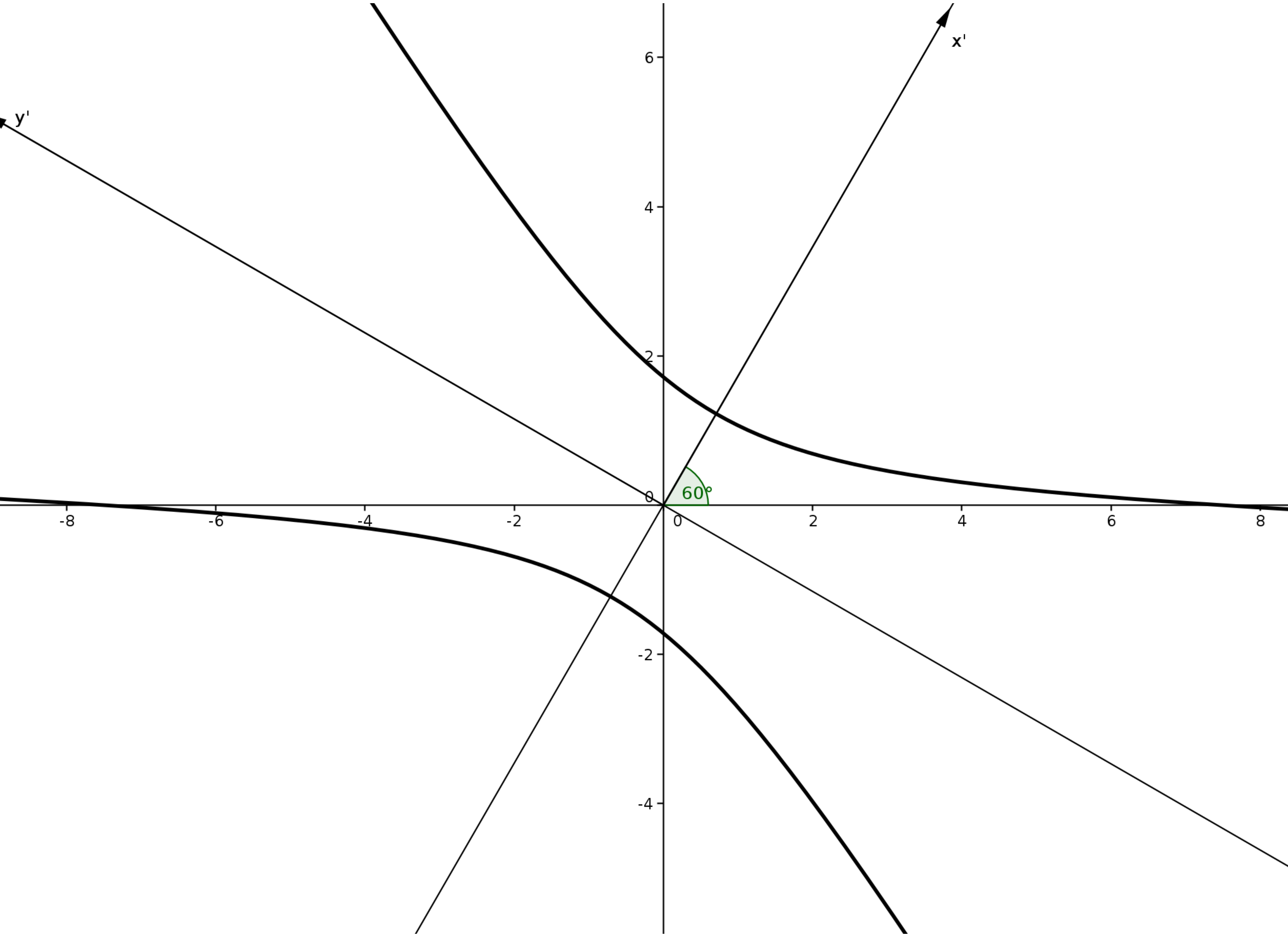
iii. $\cos\theta_2 = -\frac{12}{13}$ e $\operatorname{sen}\theta_2 = \frac{5}{13}$.

iv.

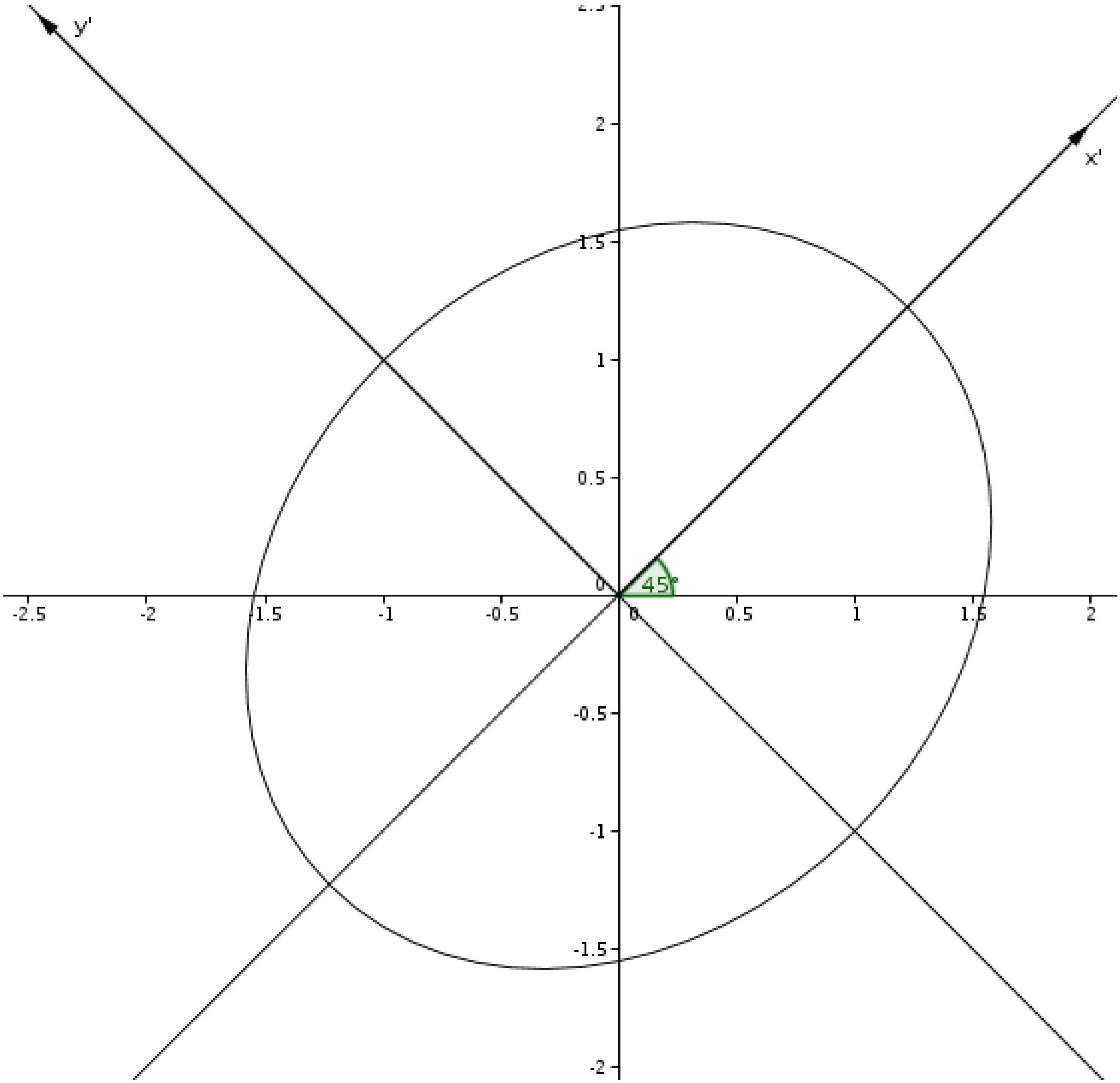
$$\begin{cases} x = -\frac{12}{13}x' - \frac{5}{13}y' \\ y = \frac{5}{13}x' - \frac{12}{13}y' \end{cases}$$

v. $y'^2 = \dfrac{x'}{13}$.

9. (a)



(b)



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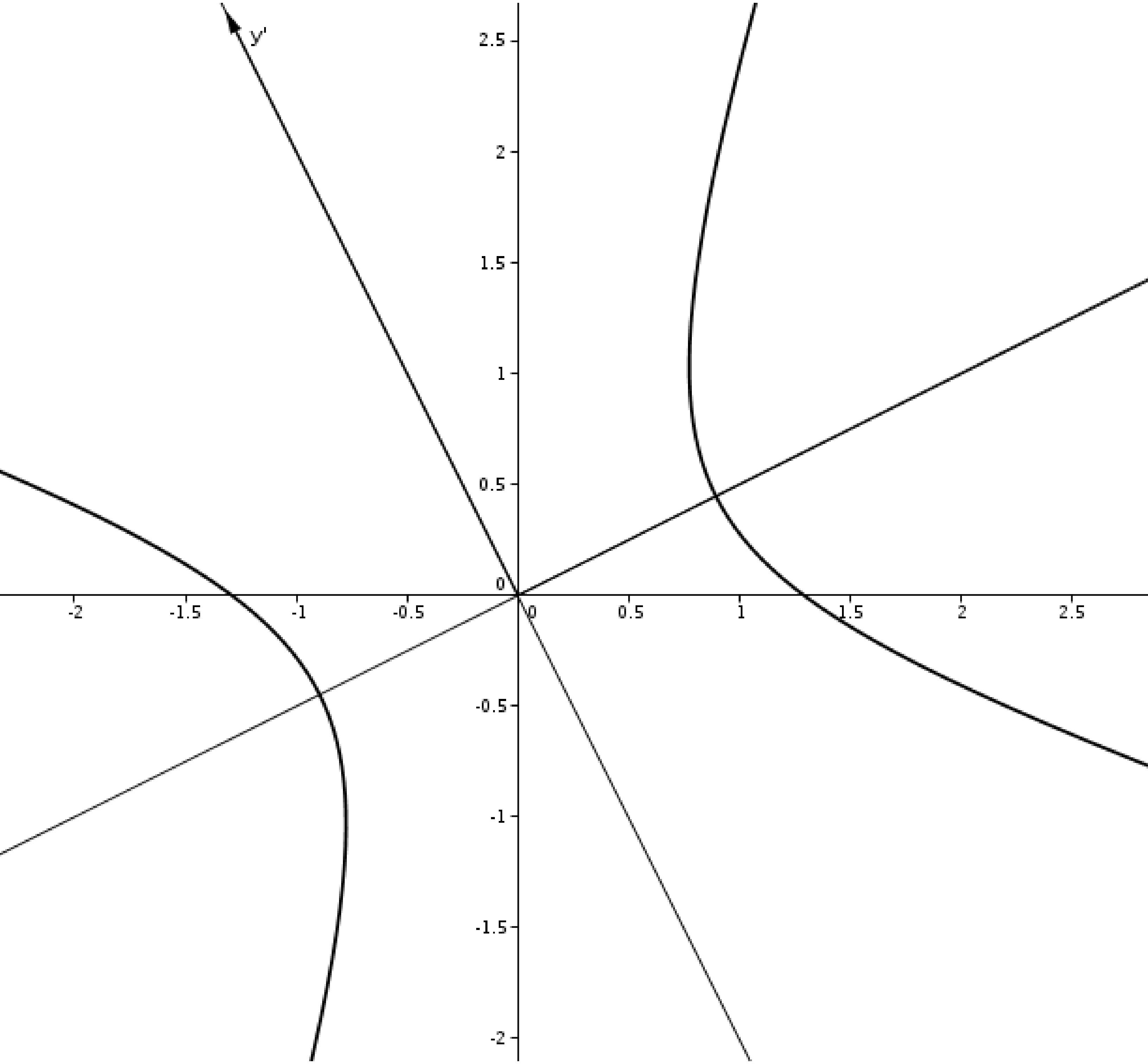
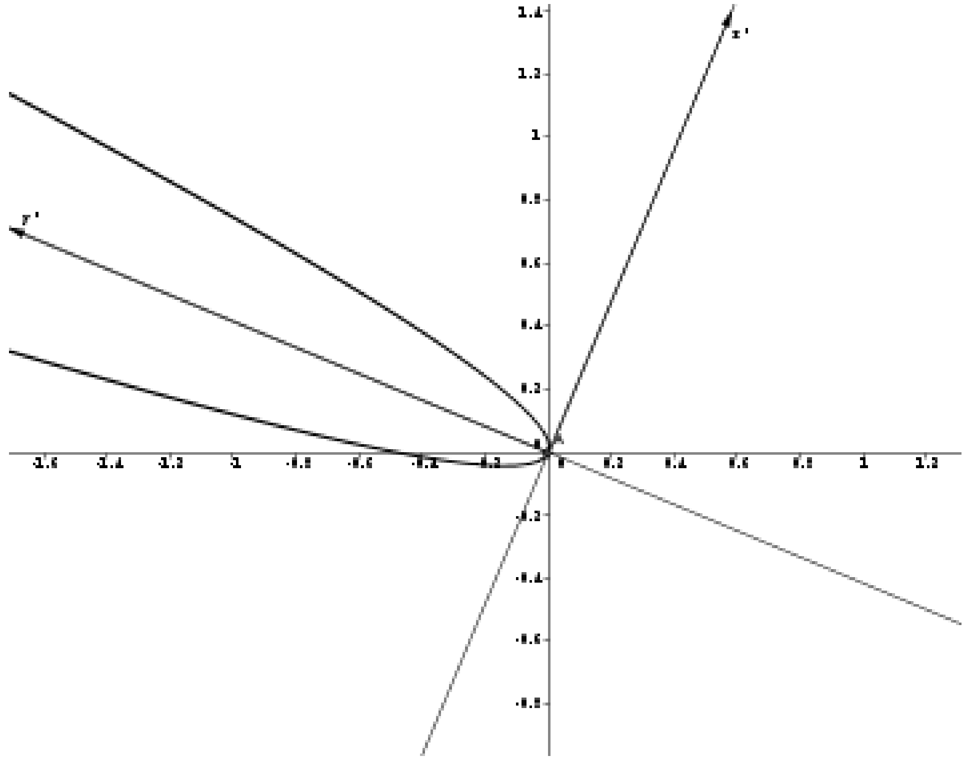


(d)


x'
y'
1.2
1