1.1 Some Basic Mathematical Models; Direction Fields

Maple Setup

We'll need the DEtools package to make DEplot available. (Recall that green, underlined text is a hyperlink that opens Help pages, other Maple worksheets, or Internet sites.) **DEplot** graphs direction fields with solution curves. End the statement with a colon to suppress printing the entire list of routines in the **DEtools** package. We'll also use the D operator for differentiating functions and diff for differentiating expressions.

```
> with(DEtools):
```

Problem 27

Draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as t approaches infinity. If this behavior depends on the initial value of y at $t = 0$, describe this dependency.

27. $y' = te^{-2t} - 2y$.

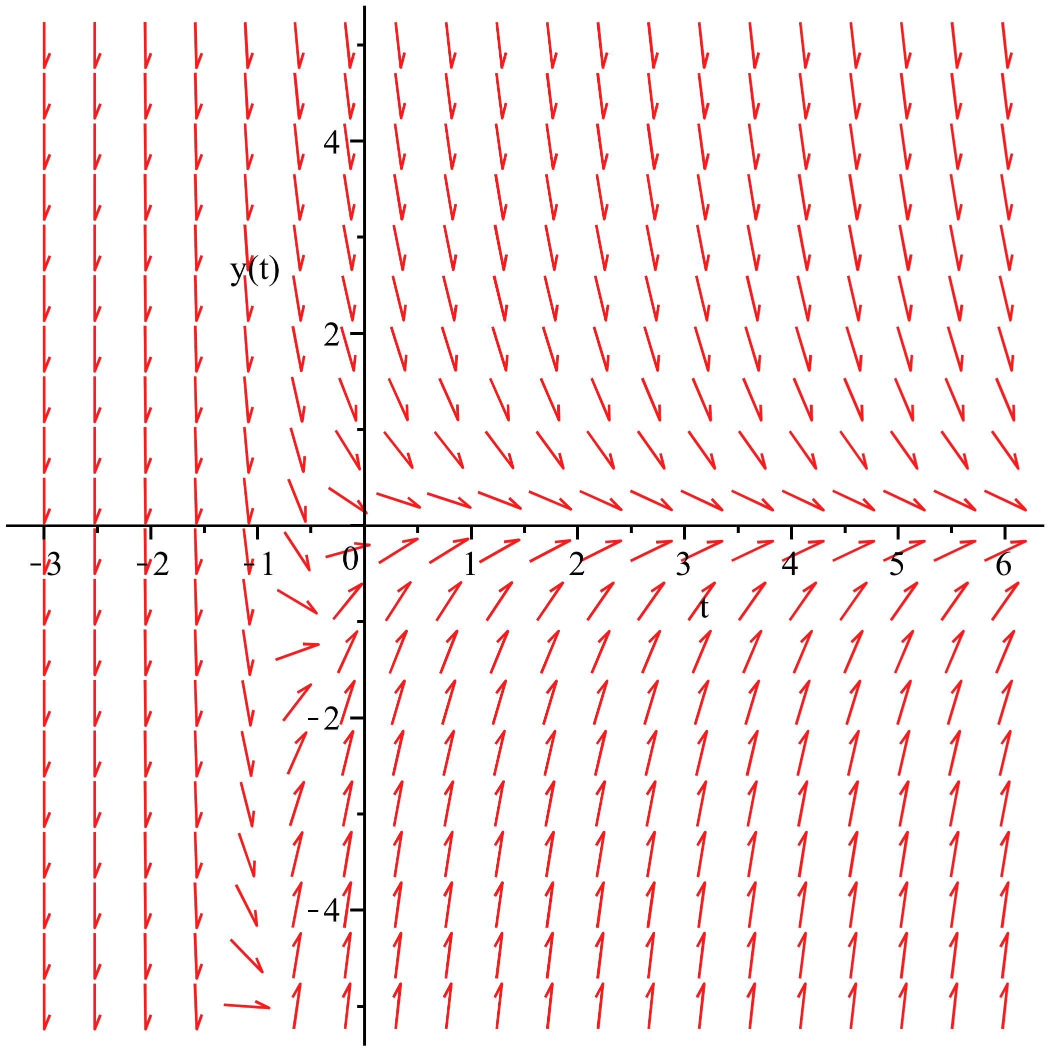
Define the differential equation, once more remembering that Maple requires us to use the full function expression $y(t)$. For variety, we'll use the derivative function diff, then y' is entered as **diff(y(t), t)**. (Remember, green text is a hyperlink that can lead to help pages.) Maple uses exp for the exponential function e^x.

```
> ode := diff(y(t), t) = t*exp(-2*t) -2*y(t);
```

$$ode := \frac{d}{dt}\, y(t) = t\, e^{-2t} - 2\, y(t)$$

Now draw a direction field using **DEplot**.

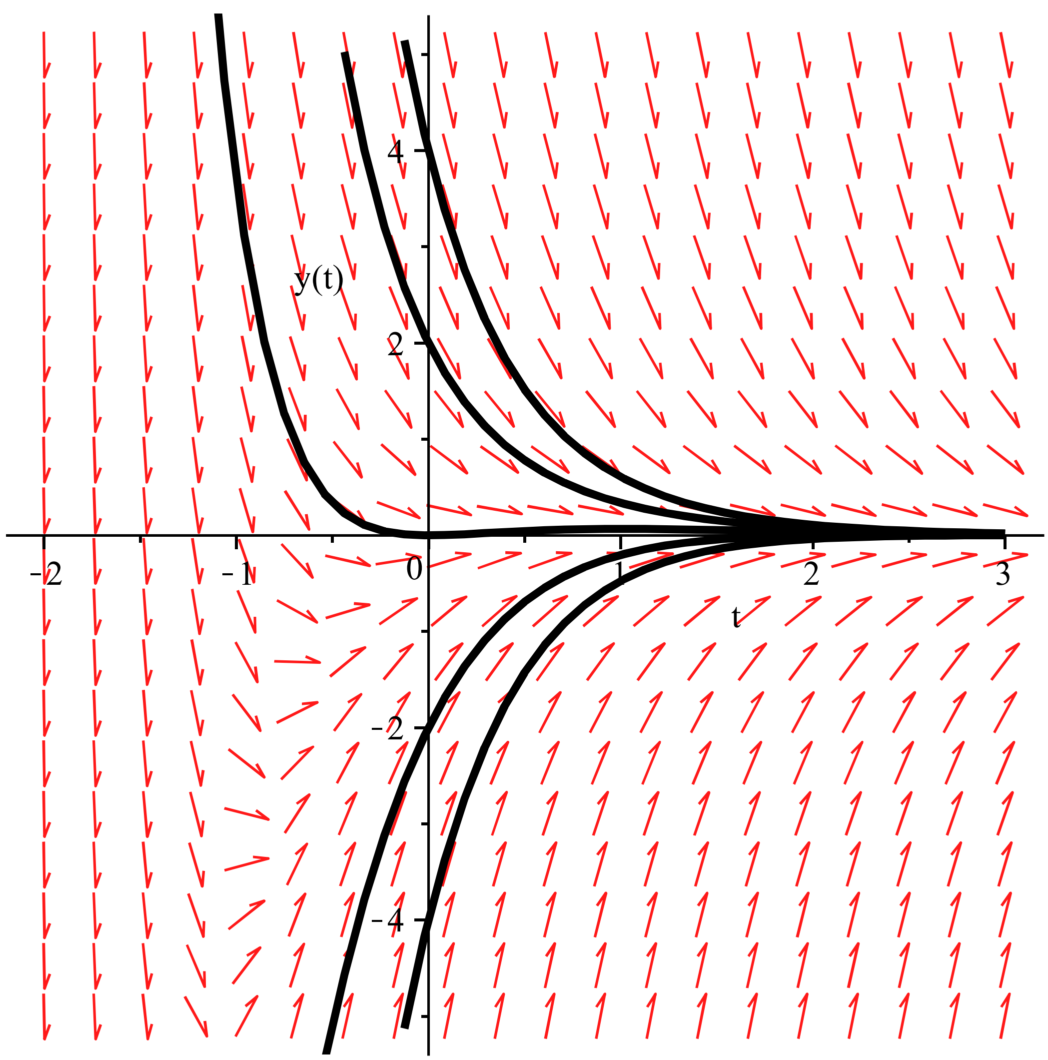
```
> DEplot(ode, y(t), t=-3..6, y=-5..5);
```



Since this equation includes terms with *t*, the behavior is more complicated. We see a pattern that indicates solution curves tend to $y = 0$. Let's investigate with the initial values y_0 = -4, -2, 0, 2, and 4. Again, put the points $[0, y_0]$ in a list.

```
> InitialValues := [[0,-4], [0,-2], [0,0], [0,2], [0,4]];
  DEplot(ode, y(t), t=-2..3, y=-5..5, InitialValues, linecolor=
  black);
```

$$InitialValues := [[0, -4], [0, -2], [0, 0], [0, 2], [0, 4]]$$



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These curves confirm our impression of the pattern shown by the direction field.