

Encontro em Geometria Diferencial  
no Rio Grande do Sul

HOMENAGEIA

O professor Dr. **Jaime Bruck Ripoll**  
pelos seus 65 anos de vida e por sua distinta carreira acadêmica.

Programação

	Segunda-feira 04/09/17	Terça-feira 05/09/17	Quarta-feira 06/09/17
09:00 – 09:45	Jorge Lira	Keti Tenenblat	Ari Aiolfi
09:45 – 10:15	Coffee Break	Coffee Break	Coffee Break
10:15 – 11:00	Marcos Dajczer	Asun Jiménez	Ivaldo Nunes
11:15 – 12:00	Marcos Cavalcante	Leonardo Bonorino	Fabiano Brito
12:00 - 14:00	Almoço	Almoço	Almoço
14:00 – 14:45	Giovanni Nunes	Ernani Ribeiro	Levi Lima
15:00 – 15:45	Cláudio Gorodski	Walcy Santos	Ezequiel Barbosa
15:45 – 16:15	Coffee Break	Coffee Break	Coffee Break
16:15 - 17:00	Almir Santos	Ruy Tojeiro	Paolo Piccione
		(20h00) Jantar Social	

## Informações úteis.

- Bem-vindo(a)s à Porto Alegre! Primeiramente, avisamos Porto Alegre tem um clima ligeiramente temperamental... Assim, as temperaturas na cidade costumam ser instáveis, podendo variar muito de um dia para o outro. É comum ocorrerem grandes amplitudes térmicas no decorrer de um mesmo dia. Durante o evento, as temperaturas previstas estão entre 15°C a 27°C. Logo, recomenda-se aos participantes que tragam pelo menos algum agasalho em caso de tempo frio.
- A cidade conta com os serviços usuais de transporte urbano (ônibus, taxi, etc), além dos serviços de transporte individual por aplicativo, como Uber e Cabify. Esses aplicativos são os mais recomendados para curtos a médios deslocamentos, por terem um bom custo/benefício e oferecerem melhor conforto e segurança, quando comparados a ônibus ou táxis comuns.
- O jantar social será na Churrascaria Barranco (Av. Protásio Alves, 1578 - Bairro Petrópolis). O valor para cada participante será de R\$90,00, que inclui: saladas, entradas diversas e o espeto corrido. Bebidas e sobremesas são cobradas a parte. Para que a churrascaria reserve um espaço adequado às nossas necessidades, pedimos a gentileza de confirmar presença até segunda (04/08) à noite ou terça (05/08) pela manhã.
- Todas as palestras do evento acontecerão no **Hotel Everest**, localizado na Rua Duque de Caxias nº1357, no Centro Histórico de Porto Alegre.

### Comitê científico

Harold Rosenberg (IMPA)  
Jaime Ripoll (UFRGS/UFSM)  
Jorge Lira (UFC)  
William Meeks (UMass Amherst)

### Comitê organizador

Álvaro Ramos (UFRGS)  
Ari Aiolfi (UFSM)  
Miriam Telichevesky (UFRGS)  
Patrícia Klaser (UFSM)

### Apoio



# Resumos

**Segunda-feira, 04 de setembro**

**09h00 – Jorge Lira (UFC)**

**Mean curvature flow solitons in warped spaces**

**Resumo/Abstract:**

We formulate a notion of self-similar solution for the mean curvature flow in warped spaces, in particular space forms. Some basic foundational results will be discussed in the sequel. This is a joint work with L. Alías (Murcia) and M. Rigoli (Milano).

**10h15 – Marcos Dajczer (IMPA)**

**Entire unbounded constant mean curvature Killing graphs**

**Resumo/Abstract:**

In this talk, we provided conditions for an entire constant mean curvature Killing graph lying inside a possible unbounded region to be necessarily a slice. This is joint work with Jorge H. Lira.

**11h15 – Marcos Petrúcio Cavalcante (UFAL)**

**Gap theorems for free boundary minimal hypersurfaces in the Euclidean ball**

**Resumo/Abstract:**

In this talk I will present some recent results about the geometry and topology of a free boundary minimal hypersurface  $x : M^n \rightarrow B^{n+1}$  in the Euclidean ball.

In a joint work with Celso Viana and Ezequiel Barbosa we prove that if  $\|x^\perp\|^2 \|A\|^2 \leq n/(n-1)$  then  $M$  is diffeomorphic to either  $\mathbb{D}^n$  or to  $\mathbb{S}^1 \times \mathbb{D}^{n-1}$ . We also discuss some recent examples and generalizations.

In a joint work with Abraão Mendes and Feliciano Vitorio we prove that if  $n \geq 4$  and  $\|A\|^2 < [n(n-2)/2(n-1)]^2$  then the space of harmonic 1-forms that are normal to  $\partial M$  is trivial. If  $n \geq 4$  and  $\|A\|^2 < n(n-2)/2(n-1)$  then the space of harmonic 1-forms that are tangential to  $\partial M$  is also trivial. These results are extensions of a recent work of Ambrosio and Nunes and give a positive answers of a question posed in their article.

**14h00 – Giovanni Nunes (UFPEL)**

**Funcional Energia: Um paralelo entre os pontos críticos dos campos de norma pontual igual a um e os de norma  $L^2$  igual a um.**

**Resumo/Abstract:**

Nesta palestra pretendo expor resultados obtidos de artigos publicado em colaboração com os professores Jaime Ripoll , Fabiano Brito, André Gomes e Pablo Chacón, envolvendo pontos críticos do funcional energia de campos onde a restrição é a de que os campos tenham norma pontual igual a um, bem como fazer um paralelo com resultados, em andamento, obtidos para este funcional quando a restrição para os campos é o da norma  $L^2$  igual a um. Ao final, pretendo apresentar algumas das propostas de investigações futuras envolvendo este tema.

**15h00 – Claudio Gorodski (USP)**

**Diameter and curvature of orbit spaces**

**Resumo/Abstract:**

Let a compact Lie group  $G$  act by isometries (non-transitively) on the unit sphere  $S^n(1)$  (with  $n \geq 2$ ). The space of orbits  $X$  is an Alexandrov space of curvature at least 1 (if  $\dim X \geq 2$ ) and diameter at most  $\pi$  with respect to the natural quotient metric. The following question of K. Grove has been investigated by several authors and remains widely open in general:

*How small can the diameter of  $X$  be?*

In this talk, we address the closely related problems:

*How curved can  $X$  be?*

and

*How curved can the  $G$ -orbits be?*

(Based on joint work with A. Lytchak (Köln) and A. B. Saturnino (UPenn).)

**16h15 – Almir Santos (UFS)**

**Solutions to the singular 2-Yamabe problem with isolated singularities**

**Resumo/Abstract:**

Given a closed Riemannian manifold  $(M, g)$  and a nonempty closed subset  $X$  in  $M$ , the singular  $k$ -Yamabe problem asks for a complete metric on the complement of  $X$  in  $M$ ,

conformal to  $g$  with constant  $k$ -curvature. The  $k$ -curvature is defined as the  $k$ -th elementary symmetric function of the eigenvalues of the Schouten tensor of a Riemannian metric. This problem is equivalent to find a solution of a PDE in  $M \setminus X$ . In this talk we will show how to use perturbation techniques and gluing methods to obtain solutions when  $X$  is a discrete set.

**Terça-feira, 05 de setembro**

**09h00 – Ketí Tenenblat (UNB)**

### **On Dupin hypersurfaces with constant Laguerre curvature**

#### **Resumo/Abstract:**

We consider proper Dupin hypersurfaces  $M^n$ ,  $n \geq 3$ , of the Euclidean space, parametrized by lines of curvature with  $n$  distinct principal curvatures. Assuming that the principal curvatures of  $M^n$  do not vanish, we obtain explicitly all such Dupin hypersurfaces with constant Laguerre curvatures. In particular, we show that they are determined by  $n$  real constants, namely,  $(n - 2)$  Laguerre curvatures and two other constants, one of them being nonzero

**10h15 – Asun Jiménez (UFF)**

### **Singularidades isoladas da equação de curvatura média prescrita no espaço de Minkowski 3-dimensional**

#### **Resumo/Abstract:**

Nesta palestra estudaremos singularidades isoladas não removíveis da seguinte EDP quase-linear, não uniformemente elíptica em duas variáveis:

$$(1 - z_y^2)z_{xx} + 2z_x z_y z_{xy} + (1 - z_x^2)z_{yy} = 2\mathcal{H}(1 - z_x^2 - z_y^2)^{3/2},$$

onde  $\mathcal{H} = \mathcal{H}(x, y, z)$  é uma função positiva e  $z = z(x, y)$  satisfaz a condição de elipticidade  $z_x^2 + z_y^2 < 1$ . As soluções desta equação têm uma interpretação geométrica, já que representam gráficos de superfícies espaciais de curvatura média  $\mathcal{H}$  no espaço de Lorentz-Minkowski  $\mathbb{L}^3$ .

Mais concretamente, consideraremos soluções elípticas  $z(x, y)$  que são  $C^2$  num certo disco pontilhado

$$\Omega = \{(x, y) : (x - x_0)^2 + (y - y_0)^2 < \rho^2\} \subset \mathbb{R}^2,$$

e que não se estendem de forma suave a través da singularidade  $(x_0, y_0)$ . Descreveremos o comportamento assintótico das soluções perto duma singularidade não removível e daremos uma classificação descrevendo o espaço moduli associado.

**11h15 – Leonardo Bonorino (UFRGS)**

## **Isolated asymptotic singularities of solutions of quasi-linear elliptic equations on a Cartan-Hadamard manifold**

### **Resumo/Abstract:**

Let  $M$  be a 2-dimensional Cartan-Hadamard manifold with sectional curvature satisfying  $-b^2 \leq K \leq -a^2 < 0$ ,  $b \geq a > 0$ . Denote by  $\partial_\infty M$  the asymptotic boundary of  $M$  and by  $\bar{M} := M \cup \partial_\infty M$  the geometric compactification of  $M$  with the cone topology. Given a finite number of points  $p_1, \dots, p_n \in \partial_\infty M$ , suppose that  $u \in C^1(M) \cap C^0(\bar{M} \setminus \{p_1, \dots, p_n\})$  is a solution of the quasi-linear elliptic equation

$$\mathcal{Q}(u) = \operatorname{div} \left( \frac{\mathcal{A}(|\nabla u|)}{|\nabla u|} \nabla u \right) = 0,$$

and  $u|_{\partial_\infty M \setminus \{p_1, \dots, p_n\}}$  extends continuously to  $p_i$ ,  $i = 1, \dots, n$ . We investigate when  $u \in C^0(\bar{M})$ . We prove this provided  $\mathcal{A}$  satisfies some structural and growth conditions, which includes the minimal graph PDE. We also generalize this result for the Hyperbolic Space  $\mathbb{H}^n$ ,  $n \geq 2$ , and show that it may be false if  $\mathcal{A}$  does not satisfy such growth condition. For instance, in  $\mathbb{H}^n$  the result does not hold for the  $p$ -Laplacian equation,  $p > 1$ .

**14h00 – Ernani Ribeiro Jr (UFC)**

## **Critical metrics of the volume functional on compact manifolds with boundary**

### **Resumo/Abstract:**

We talk about the space of smooth Riemannian structures on compact manifolds with boundary that satisfies a critical point equation associated with a boundary value problem, for simplicity,  $V$ -static metrics. It is known that  $V$ -static metrics are important in understanding the interplay between volume and scalar curvature. It arises from the modified problem of finding stationary points for the volume functional on the space of metrics whose scalar curvature is equal to a given constant.

We provide an estimate to the area of the boundary of critical metrics of the volume functional on compact three-manifolds. Moreover, we obtain a Böchner type formula which enables us to classify critical metrics of the volume functional on a compact three-manifold with positive scalar curvature. In addition, we classify three-dimensional  $V$ -static spaces with non-negative sectional curvature.

**15h00 – Walcy Santos (UFRJ)**

## **The functional $L^2$ -norm of the second fundamental**

### **Resumo/Abstract:**

We consider  $f : M^n \rightarrow N^{n+p}$  a isometric immersion of a Riemannian manifold  $M$  into  $N$  and denote by  $\alpha$  its second fundamental form. We define the following functional

$$\Phi(M) = \int_M \|\alpha\|^2 d\mu,$$

and view it as a functional defined over the space of all isometric immersions of  $M$  into  $N$ . We will discuss some properties of the critical points of this functional.

**16h15 – Ruy Tojeiro (UFSCar)**

## **Hypersurfaces of two space forms and conformally flat hypersurfaces**

### **Resumo/Abstract:**

In this talk we first report on recent work in collaboration with S. Canevari on the problem of determining the hypersurfaces  $f : M^n \rightarrow \mathbb{Q}^{n+1}(c)$  with dimension  $n \geq 3$  of a space form of dimension  $n+1$  and constant curvature  $c$  for which there exists another isometric immersion  $\tilde{f} : M^n \rightarrow \mathbb{Q}^{n+1}(\tilde{c})$  with  $\tilde{c} \neq c$ . In particular, we present a characterization of the solutions of this problem that have dimension  $n = 3$  and three distinct principal curvatures. We show that these are closely related to conformally flat hypersurfaces of  $\mathbb{Q}^4(c)$  with three distinct principal curvatures, and we obtain a similar characterization of the latter that improves a theorem by Hertrich-Jeromin. We also show how these characterizations can be used to develop a Ribaucour transformation for both classes of hypersurfaces, which gives a process to produce a family of new elements of those classes starting from a given one and a solution of a linear system of PDE's. This enables us to construct explicit new examples of hypersurfaces in both classes.

Then we discuss a joint work with Carlos do Rei Filho on minimal conformally flat hypersurfaces  $f : M^3 \rightarrow \mathbb{R}^4$  with three distinct principal curvatures, in which we show that, besides the cone over the Clifford torus in  $\mathbb{S}^3 \subset \mathbb{R}^4$ , there exists precisely a one-parameter family of (congruence classes of) minimal isometric immersions  $f : M^3 \rightarrow \mathbb{R}^4$  with three distinct principal curvatures of simply-connected conformally flat Riemannian manifolds. If time permits, we will finally discuss how some of the previous results can be used to classify conformally flat hypersurfaces  $f : M^3 \rightarrow \mathbb{R}^4$  with three distinct principal curvatures and constant scalar curvature.

**Quarta-feira, 06 de setembro**

**09h00 – Arì Aiolfi (UFSM)**

**The Dirichlet problem for the minimal hypersurface equation with Lipschitz continuous boundary data on bounded domains of a Riemannian manifold**

**Resumo/Abstract:**

Joint work with G. Nunes, L. Sauer, R. B. Soares. Given a bounded, non mean convex  $C^2$ -domain  $\Omega \subset M$ , where  $M$  is an arbitrary complete Riemannian manifold, we search for smallness conditions on the boundary data for which the Dirichlet problem for the minimal hypersurface equation is solvable. We obtain an extension to Riemannian manifolds of an existence result of G. H. Williams (J. Reine Angew. Math., 354:123-140, 1984).

**10h15 – Ivaldo Nunes (UFMA)**

**Superfícies com curvatura média constante estáveis em domínios convexos do espaço Euclidiano**

**Resumo/Abstract:**

Nesta palestra, trataremos do problema de bordo livre para superfícies com curvatura média constante (CMC) em domínios convexos no espaço Euclidiano tridimensional. Provaremos que toda superfície CMC estável com bordo livre em um domínio convexo com geometria suficientemente próxima da bola unitária é homeomorfa a um disco ou um anel. Este fato, juntamente com um resultado prévio devido a A. Ros e E. Vergasta, implica em uma completa classificação destas superfícies na bola unitária.



**11h15 – Fabiano Brito (UFABC)**

**Uma formula integral para fluxos em hipersuperfícies do Euclidiano. Um limite inferior topológico para a energia do fluxo.**

**Resumo/Abstract:**

Trabalho conjunto com Adriana Nicoli e Ícaro Gonçalves. Usando métodos totalmente elementares, podemos deduzir novas fórmulas integrais para as curvaturas das folheações (ou distribuições) de codimensão 1 em hipersuperfícies fechadas de dimensão ímpar que generalizam formulas do mesmo tipo já conhecidas para os espaços de curvatura constante (—, Langevin, Rosenberg). O grau da aplicação normal intervira na invariância das integrais. Derivaremos algumas consequências concernindo folheações com grau de nulidade da 2a forma fundamental das folhas maior ou igual a 2 (em particular as folheações totalmente geodésicas). É também possível arbitrar um limite inferior para a energia de campos unitários onde intervém a topologia da imersão da hipersuperfície (grau da ap. normal) no espaço Euclidiano. Alguns tópicos sobre construção de imersões de hipersuperfícies com graus arbitrários da aplicação normal serão discutidos segundo os trabalhos de J. Milnor e S. Smale.

**14h00 – Levi Lima (UFC)**

**A probabilistic proof of the Gauss-Bonnet formula for manifolds with boundary**

**Resumo/Abstract:**

We present a simple probabilistic proof of the Gauss-Bonnet formula for compact Riemannian manifolds with boundary, which adapts to this setting an argument due to Hsu in the closed case. The new technical ingredient is the Feynman-Kac formula for differential forms satisfying absolute boundary conditions recently proved by the speaker. Combined with the so-called supersymmetric approach to index theory, this leads to a path integral representation for the Euler characteristic of the manifold in terms of normally reflected Brownian motion whose short time asymptotics clarifies the role played by the shape operator in determining the boundary contribution to the formula. As a consequence, we obtain the expected local Gauss-Bonnet formula which upon integration yields the desired global result.

**15h00 – Ezequiel Barbosa (UFMG)**

## **Resultados de gap para área de subvariedades mínimas free-boundary em bolas de formas espaciais**

### **Resumo/Abstract:**

Nessa palestra, vamos discutir resultados de gap envolvendo a área de  $k$ -subvariedades mínimas free-boundary na bola unitária Euclidiana  $B^n$ . Mais precisamente, vamos mostrar que existe um  $\varepsilon(k, n)$  tal que a única  $k$ -subvariedade mínima free-boundary  $\Sigma$  na bola Euclidiana  $B^n$  com área  $\varepsilon(n)$ -próxima da área do disco flat é o próprio disco flat. Além disso, vamos discutir resultados de gap para a energia de Willmore para superfícies CMC's free-boundary em bolas de formas espaciais  $W^3$ . Este é um trabalho em conjunto com C. Viana.

**16h15 – Paolo Piccione (USP)**

## **Teichmüller theory, collapse of flat manifolds and applications to the Yamabe problem**

### **Resumo/Abstract:**

Using certain non-uniqueness results for the Yamabe problem as motivation, I will describe deformations of compact flat manifolds and orbifolds. Flat orbifolds (resp., manifolds) are quotients of Euclidean spaces by crystallographic (resp., torsion-free crystallographic) groups. I will explain the basic structure of these objects and describe the space of deformations of flat metrics on them (Teichmüller space), showing that flat manifolds can always be deformed, while flat orbifolds may be rigid. I will also describe the boundary of the moduli space; showing that limits of flat manifolds are flat orbifolds and, conversely, that every flat orbifold is the limit of flat manifolds. This is joint work with Renato Bettiol and Andrzej Derdzinski.