

# Nonlinear structures: explosive, soliton and shock in a quantum electron-positron-ion magnetoplasma

R. Sabry<sup>1,a</sup>, W. M. Moslem<sup>1,b</sup>, F. Haas<sup>1,c</sup>, S. Ali<sup>2,d</sup> and P. K. Shukla<sup>1,e</sup>

<sup>1</sup>*Institut für Theoretische Physik IV,*

*Fakultät für Physik und Astronomie,*

*Ruhr-Universität Bochum, D-44780 Bochum, Germany and*

<sup>2</sup>*National Center for Physics, Quaid-I-Azam University Campus, Islamabad, Pakistan*

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## Abstract

Theoretical and numerical studies are performed for the nonlinear structures (explosive, solitons and shock) in quantum electron-positron-ion magnetoplasmas. For this purpose, the reductive perturbation method is employed to the quantum hydrodynamical equations and the Poisson equation, obtaining extended quantum Zakharov-Kuznetsov equation. The latter has been solved using the generalized expansion method to obtain a set of analytical solutions, which reflect the possibility of the propagation of various nonlinear structures. The relevance of the present investigation to the white dwarfs is highlighted.

## I. INTRODUCTION

Numerous investigations [1, 2, 3, 4, 5] relating to wave phenomena, have been studied in dense quantum plasmas, are of fundamental importance for understanding collective interactions in superdense astrophysical environments [6], in high intense laser-solid density experiments [7], in ultracold plasmas [8], in microplasmas [9], and in micro-electronic devices [10]. New characteristics of quantum plasma arise due to the pressure law describing the fermionic behavior of the charged carriers, quantum forces associated with the electron tunneling, as well as the Bohr magnetization involving the electron 1/2 spin. The quantum Bohm potential produces modifications in the dispersions of collective modes at quantum scales. The latter are strongly effected by the plasma number densities and Fermi temperatures. It is well-known that quantum mechanical effects become relevant when the thermal de Broglie wavelength of the charged particles is equal or larger than the average interparticle distance. In particular, quantum behavior of the electrons reaches much easily due to less mass compared to ions.

In recent years, many theoretical and numerical analysis [11, 12, 13, 14, 15] have been carried out to investigating the new features of plasmas with quantum corrections by using both the Schrödinger-Poisson and the Wigner-Poisson systems. In this context, Manfredi [11] reported different approaches to model the collisionless electrostatic dense quantum plasmas. Haas *et al.* [12] investigated the linear and nonlinear properties of the quantum ion-acoustic (QIA) waves in dense quantum plasmas by employing the quantum hydrodynamical (QHD) equations for inertialess electrons and mobile ions. They examined that the quantum Bohm potential modifies the linear wave dispersion and affects strongly the QIA solitary waves. Shukla and Eliasson [13] presented the numerical study of the dark solitons and vortices in quantum electron plasmas. Moslem *et al.* [14] investigated the quantum dust-acoustic double layers in a multi-species quantum dusty plasma. It was found that both compressive and rarefactive double layers can only exist for positively charged dust particles. Later, Ali *et al.* [16] studied the QIA waves in a three-component plasma, comprised of electrons, positrons, and ions. They employed the reductive perturbation method and pseudo-potential approach for the small and arbitrary amplitude nonlinear QIA waves, respectively. It was shown that the amplitude and width are significantly altered due to the quantum statistics and quantum tunneling effects. Misra *et al.* [17] considered the nonlinear propagation of electron-acoustic

waves in a nonplanar quantum plasma, consisting of two groups of electrons: the inertial cold electrons and inertialess hot electrons as well as the stationary ions. They obtained the bright and dark solitons depending strongly upon the presence of cold electrons.

The laboratory and dense astrophysical quantum plasmas can be confined by an external magnetic field. Therefore, the effect of the magnetic field has to be taken into account, especially for astrophysical observations (such as white dwarfs, neutron stars, magnetars, etc.) where the high magnetic field plays an important role in the formation and stability of the existing waves. Several authors have considered the effect of magnetic field in different quantum plasma models. For example, Haas [18] introduced a three-dimensional QHD model for dense magnetoplasmas and established the conditions for an equilibrium in the ideal quantum magnetohydrodynamics (QMHD). Ali *et al.* [19] employed the QMHD equations presenting a fully nonlinear theory for ion-sound waves in a dense Fermi magnetoplasma. It was revealed that only subsonic ion-sound solitary waves may exist. Shukla and Stenflo [20] derived the dispersive shear Alfvén waves in a quantum magnetoplasma, incorporating the strong electron and positron density fluctuations. The shear Alfvén modes acquire additional dispersion due to quantum corrections. Later, Ali *et al.* [21] have been carried out for the low-frequency electrostatic drift-like waves in a nonuniform collisional quantum magnetoplasma. It was shown that the modes become unstable and can cause cross-field anomalous ion-diffusion.

Three decades ago, Zakharov and Kuznetsov [22] derived an equation for nonlinear ion-acoustic waves in a magnetized plasma containing cold ions and hot isothermal electrons. The Zakharov-Kuznetsov (ZK) equation has also been derived for different physical systems and scenarios [23, 24]. Nonlinear wave solution for ZK equation can produce an instability in a three-dimensional system as discussed in Refs. [25, 26]. Moslem *et al.* [27] extended the work for a three-dimensional nonlinear ion-acoustic waves in a quantum magnetoplasma, highlighting the bending instability of the solitary wave solution of the quantum ZK equation. Recently, Masood and Mushtaq [28] studied obliquely propagating electron-acoustic waves in a two-electron population quantum magnetoplasma and examining the effects of nonlinearity at quantum scales.

In the present paper, we shall investigate the possible nonlinear structures (soliton, explosive and shock pulses) of the QIA waves in a collisionless electron-positron-ion magnetoplasma using the QHD equations. By means of computational investigations, we examine

the effect of the positron concentration, the quantum diffraction and the quantum statistical effects on the profiles of the nonlinear excitations. The paper is organized as follows: The basic equations governing the dynamics of the nonlinear QIA waves are presented and the extended quantum ZK equation describing the system is derived in Sec II. In Sections III and IV, we apply the generalized expansion method to solve the extended quantum ZK equation. A set of analytical solutions is obtained, and then used to investigate numerically the effect of positrons and the quantum parameters on the nonlinear excitations. The results are summarized in section V.

## II. BASIC EQUATIONS AND DERIVATION OF THE EXTENDED QUANTUM ZK EQUATION

We consider a dense magnetoplasma whose constituents are the electrons, positrons, and singly charged positive ions. The plasma is confined in an external magnetic field  $\mathbf{H}_0 = H_0 \hat{\mathbf{z}}$ , where  $\hat{\mathbf{z}}$  is the unit vector along the  $z$ -axis and  $H_0$  is the strength of the magnetic field. We assume that the quantum plasma satisfies the condition  $T_{F_{e,p}} \gg T_{F_i}$ , and obeys the electron/positron pressure law  $P_{e,p} = m n_{e,p}^3 V_{F_{e,p}}^2 / 3n_{e,p0}^2$ , where  $V_{F_{e,p}} = (2K_B T_{F_{e,p}} / M)^{1/2}$  is the electron/positron Fermi thermal speed,  $K_B$  is the Boltzmann constant,  $T_{F_{e,p}}$  ( $T_{F_i}$ ) is the electron/positron (ion) Fermi temperature,  $M$  is the electron and positron mass,  $n_{e,p}$  is the electron/positron number density, with the equilibrium value  $n_{e,p0}$ . The nonlinear propagation of the QIA waves is governed by the dimensionless hydrodynamics equations as

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0, \quad (1)$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + \mathbf{u}_i \cdot \nabla \mathbf{u}_i = -\nabla \phi + \mathbf{u}_i \times \hat{\mathbf{z}}, \quad (2)$$

$$\Omega \nabla^2 \phi = n_e - n_p - n_i, \quad (3)$$

$$n_e = \mu_e \left( 1 + 2\phi + H_e^2 \frac{\nabla^2 \sqrt{n_e}}{\sqrt{n_e}} \right)^{\frac{1}{2}}, \quad (4)$$

and

$$n_p = \mu_p \left( 1 - 2\sigma\phi + \sigma H_e^2 \frac{\nabla^2 \sqrt{n_p}}{\sqrt{n_p}} \right)^{\frac{1}{2}}, \quad (5)$$

where  $n_i$ ,  $\mathbf{u}_i$ , and  $\phi$  are the ion number density, the ion fluid velocity, and the electrostatic potential, respectively. Since, the ion mass is much larger than the electron/positron mass,

one can ignore the quantum effects of the ions in Eq. (2). The statistical and diffraction effect for the system can be seen through the nondimensional parameters  $\sigma(= T_{Fe}/T_{Fp})$  and  $H_e(= eH_0\hbar/2c\sqrt{M_iM}K_B T_{Fe})$ , respectively, where  $\hbar$  is the Planck constant divided by  $2\pi$ ,  $M_i$  ( $M$ ) is the ion (electron/positron) mass, and  $c$  is the speed of light in vacuum. Here,  $\Omega(= \omega_{ci}/\omega_{pi})$ , where  $\omega_{ci} (= eH_0/m_i c)$  and  $\omega_{pi} (= \sqrt{4\pi e^2 n_{i0}/M_i})$  are the ion gyrofrequency and the ion plasma frequency, respectively.  $n_{i0}$  is the equilibrium ion density. Equations (4) and (5) reveal that the electrons and positrons do not follow the Boltzmann law contrary to the classical plasma. The physical quantities appearing in Eqs. (1)–(5) have been appropriately normalized:  $n_{e,i,p} \rightarrow n_{e,i,p}/n_{i0}$ ,  $\mathbf{u}_i \rightarrow \mathbf{u}_i/C_s$ ,  $t \rightarrow t\omega_{ci}$ ,  $\nabla \rightarrow \nabla\rho_s$ , and  $\phi \rightarrow e\phi/2K_B T_{Fe}$ , where  $\rho_s(= C_s/\omega_{ci})$  is the ion-sound Fermi gyroradius and  $C_s(= \sqrt{2K_B T_{Fe}/M_i})$  is the ion-sound Fermi speed.

Before going to the nonlinear developments, it is necessary to examine the condition for neglecting the source term in the continuity equation due to annihilation of plasma species. The details are given in the Appendix.

To investigate the propagation of QIA waves, we expand the dependent variables  $n_{e,i,p}$ ,  $\mathbf{u}_i$ , and  $\phi$  about their equilibrium values in power of  $\epsilon$ ,

$$\begin{aligned}
n_i &= 1 + \epsilon n_{i1} + \epsilon^2 n_{i2} + \epsilon^3 n_{i3} + \dots, \\
n_{e,p} &= \mu_{e,p} + \epsilon n_{e,p1} + \epsilon^2 n_{e,p2} + \epsilon^3 n_{e,p3} + \dots, \\
u_{ix,y} &= \epsilon^2 u_{ix,y1} + \epsilon^3 u_{ix,y2} + \epsilon^4 u_{ix,y3} + \dots, \\
u_{iz} &= \epsilon u_{iz1} + \epsilon^2 u_{iz2} + \epsilon^3 u_{iz3} + \dots, \\
\phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \dots,
\end{aligned} \tag{6}$$

where  $\epsilon$  is a keeping order parameter proportional to the amplitude of the perturbation. Following the reductive perturbation method [29], we express the independent variables into a moving frame in which the nonlinear wave moves at a phase-speed of  $\lambda$  (normalized with the ion-sound Fermi speed  $C_s$ ) as

$$X = \epsilon x, \quad Y = \epsilon y, \quad Z = \epsilon(z - \lambda t) \quad \text{and} \quad T = \epsilon^3 t. \tag{7}$$

The neutrality condition at equilibrium reads  $\mu_e = 1 + \mu_p$ , where  $\mu_e = n_{e0}/n_{i0}$  and  $\mu_p = n_{p0}/n_{i0}$ . Substituting (6) and (7) into Eqs. (1)–(5), we obtain the lowest-order in  $\epsilon$  as

$$\begin{aligned}
n_{i1} &= \frac{1}{\lambda^2} \phi_1, & u_{ix1} &= -\frac{\partial \phi_1}{\partial Y}, \\
u_{iy1} &= \frac{\partial \phi_1}{\partial X}, & u_{iz1} &= \frac{1}{\lambda} \phi_1, \\
n_{e1} &= \mu_e \phi_1, & n_{p1} &= -\sigma \mu_p \phi_1,
\end{aligned} \tag{8}$$

along with the phase speed rule

$$\lambda = \left( \frac{1}{1 + \mu_p(1 + \sigma)} \right)^{1/2}. \tag{9}$$

It is clear here that the phase speed  $\lambda$  of the QIA waves is affected by the quantum statistical effect and by the positron concentration  $\mu_p$ . To the next-order in  $\epsilon$ , we have

$$\begin{aligned}
n_{i2} &= \frac{4}{3\lambda^4} \phi_1^2 + \frac{1}{\lambda^2} \phi_2, & u_{ix2} &= \lambda \frac{\partial^2 \phi_1}{\partial X \partial Z} - \frac{\partial \phi_2}{\partial Y}, \\
u_{iy2} &= \lambda \frac{\partial^2 \phi_1}{\partial Y \partial Z} + \frac{\partial \phi_2}{\partial X}, & u_{iz2} &= \frac{1}{2\lambda^3} \phi_1^2 + \frac{1}{\lambda} \phi_2, \\
n_{e2} &= -\frac{\mu_e}{2} (\phi_1^2 - 2\phi_2), & n_{p2} &= -\frac{\sigma \mu_p}{2} (\sigma \phi_1^2 + 2\phi_2),
\end{aligned} \tag{10}$$

while the Poisson equation gives

$$Q \phi_1^2 = 0, \tag{11}$$

where

$$Q = \frac{[(\sigma^2 - 1) \mu_p \lambda^4 - \lambda^4 - 3]}{2\lambda^4}.$$

Since  $\phi_1 \neq 0$ , therefore  $Q$  should be at least of the order of  $\epsilon$ . Therefore,  $Q \phi_1^2$  becomes of the order of  $\epsilon^3$ ; so it should be included in the next order of the Poisson equation. The next-order in  $\epsilon$  gives a system of equations. Solving this system with the aid of Eqs. (8)-(10), we finally obtain the extended quantum ZK equation as

$$\frac{\partial \varphi}{\partial T} + (A \varphi + B \varphi^2) \frac{\partial \varphi}{\partial Z} + C \frac{\partial^3 \varphi}{\partial Z^3} + D \frac{\partial}{\partial Z} \left( \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) \varphi = 0, \tag{12}$$

where we have replaced  $\phi_1$  by  $\varphi$  for simplicity. The nonlinear and dispersion coefficients are

given, as

$$\begin{aligned}
A &= \frac{\lambda^4 + 3 - (\sigma^2 - 1) \mu_p \lambda^4}{2\lambda}, \\
B &= -\frac{3((\sigma^3 + 1) \mu_p \lambda^6 + \lambda^6 - 5)}{4\lambda^3}, \\
C &= \frac{1}{8} \lambda^3 (4\Omega - (\mu_p + 1) H_e^2 - \sigma^2 H_e^2 \mu_p), \\
D &= C + \frac{1}{2} \lambda^3
\end{aligned}$$

The extended quantum ZK equation (12) constitutes the final outcome of this model. The anticipated balance between dispersion and nonlinearity (which contain the quantum mechanical effects) within the extended quantum ZK equation may give rise to different nonlinear structures. Some of these solutions will recover in the next section.

### III. EXACT SOLUTIONS OF THE EXTENDED QUANTUM ZK EQUATION

To obtain the possible analytical solutions of Eq. (12), we assume that

$$\xi = L_X X + L_Y Y + L_Z Z - \vartheta T, \quad (13)$$

where  $L_X$ ,  $L_Y$  and  $L_Z$  are the direction cosines and  $\vartheta$  is the QIA wave speed to be determined later. Using (13) into (12), we obtain

$$-\vartheta \varphi' + A_0 \varphi \varphi' + B_0 \varphi^2 \varphi' + \gamma \varphi''' = 0, \quad (14)$$

where  $A_0 = AL_Z$ ,  $B_0 = BL_Z$  and  $\gamma = CL_Z^3 + DL_Z(L_X^2 + L_Y^2)$ . According to the generalized expansion method [30] the solution of Eq. (14) can represent by

$$\varphi = a_0 + a_1 \omega, \quad (15)$$

with

$$\frac{d\omega}{d\xi} = k (c_0 + c_1 \omega + c_2 \omega^2 + c_3 \omega^3 + c_4 \omega^4)^{1/2}, \quad (16)$$

where  $a_0, a_1, c_0, c_1, c_2, c_3$  and  $c_4$  are arbitrary constants to be determined later and  $k = \pm 1$ . Substituting Eq. (15) into Eq. (14) and making use of Eq. (16), we obtain a polynomial equation in  $\omega$ . Equating the coefficients of different powers of  $\omega$ , we obtain an overdetermined

system of algebraic equations which can be solved with the help of symbolic manipulation package *Mathematica* to give three Jacobi elliptic doubly periodic type solutions as

$$\varphi = -\frac{A_0}{2B_0} + k\sqrt{\frac{6\gamma c_2 m^2}{B_0(2m^2-1)}} \operatorname{cn}\left(\sqrt{\frac{c_2}{(2m^2-1)}}\xi\right),$$

$$\text{with } c_0 = -\frac{c_2^2 m^2 (1-m^2)}{c_4 (2m^2-1)^2}, \quad c_2 > 0, \quad c_4 < 0, \quad (17)$$

$$\varphi = -\frac{A_0}{2B_0} + k\sqrt{\frac{6\gamma c_2}{B_0(2-m^2)}} \operatorname{dn}\left(\sqrt{\frac{c_2}{(2-m^2)}}\xi\right),$$

$$\text{with } c_0 = \frac{c_2^2 (1-m^2)}{c_4 (2-m^2)^2}, \quad c_2 > 0, \quad c_4 < 0, \quad (18)$$

and

$$\varphi = -\frac{A_0}{2B_0} + k\sqrt{\frac{6\gamma c_2 m^2}{B_0(m^2+1)}} \operatorname{sn}\left(\sqrt{\frac{-c_2}{(m^2+1)}}\xi\right),$$

$$\text{with } c_0 = \frac{c_2^2 m^2}{c_4 (m^2+1)^2}, \quad c_2 < 0, \quad c_4 > 0, \quad (19)$$

where  $m$  is a modulus of the Jacobian elliptic function and  $c_1 = c_3 = 0$ . As  $m \rightarrow 1$ , the Jacobi doubly periodic solutions (17) and (18) degenerate to the bell-shaped solitary wave

$$\varphi = -\frac{A_0}{2B_0} + k\sqrt{\frac{6\gamma c_2}{B_0}} \operatorname{sech}(\sqrt{c_2}\xi), \quad (20)$$

where the arbitrary constant  $c_0$  vanishes. Again, as  $m \rightarrow 1$  the solution (19) can degenerate to the kink-type wave solution

$$\varphi = -\frac{A_0}{2B_0} + k\sqrt{\frac{3\gamma c_2}{B_0}} \tanh\left(\sqrt{\frac{-c_2}{2}}\xi\right), \quad (21)$$

where  $c_0 = c_2^2/4c_4$ . In the solutions (17)-(21), the QIA wave speed  $\vartheta = \frac{1}{2}(-A_0^2/2B_0 + 2\gamma c_2)$  where  $c_2 \neq A_0^2/4\gamma B_0$ .

Furthermore, the generalized expansion method provides us with further analytical solutions of the extended quantum ZK equation (12) as

$$\varphi = -\frac{2c_2}{c_3 + k\sqrt{c_3^2 - 4c_2c_4} \cosh(2\sqrt{c_2}\xi)},$$

$$\text{with } c_0 = c_1 = 0, \quad c_2 = \frac{\vartheta}{\gamma}, \quad c_3 = -\frac{A_0}{3\gamma}, \quad c_4 = -\frac{B_0}{6\gamma}, \quad (22)$$



and

$$\varphi = -\frac{A_0}{2B_0} \left[ 1 + k \coth \left( \sqrt{\frac{-A_0^2}{24\gamma B_0}} \xi \right) \right], \quad \text{with } \vartheta = -\frac{A_0^2}{6B_0} \quad \text{and } B_0 < 0. \quad (23)$$

#### IV. PARAMETRIC ANALYSIS FOR WHITE DWARFS

It is clear that the propagation speed of the QIA wave is modified by the effect of the quantum statistical effect  $\sigma$  and by the presence of positrons  $\mu_p$ . As  $\sigma$  and  $\mu_p$  increase, the propagation speed of the QIA wave will decrease. The dependence of the nonlinear structures amplitude and width on the equilibrium positron number density ( $\mu_p$ ) and quantum effects  $\sigma$  and  $H_e$  is more perplex. First, it is important to note that changing  $\mu_p$  leads to a change in the phase-speed ( $\lambda$ ) of the QIA waves [see Eq. (9)], as well as the electron concentration (via the charge-neutrality condition  $\mu_e = 1 + \mu_p$ ). Since the electron (positron) Fermi temperature depends upon the equilibrium electron (positron) number density, it can also be affected by  $\mu_p$  through the charge-neutrality condition. As a result, the quantum statistical ( $\sigma$ ) and diffraction ( $H_e$ ) effects will vary with the positron concentration  $\mu_p$ .

Based upon the above findings, we shall now investigate the effects of the relevant physical quantities, namely the positron concentration  $\mu_p$  on the profiles of the QIA nonlinear structures. We have used, as a starting point, a typical set of plasma parameter values for white dwarfs [11] (in the absence of positrons;  $\mu_p = 0$ ), namely:  $n_{e0} = 4 \times 10^{28} \text{ cm}^{-3}$ ,  $T_{Fe} = 4.9 \times 10^8 \text{ K}$ ,  $\omega_{ci} = 1.88 \times 10^{16} \text{ s}^{-1}$  and  $\omega_{pi} = 2.63 \times 10^{17} \text{ s}^{-1}$ . However, once the positrons species density is determined, the values of  $T_{Fe}$ ,  $\lambda$  and  $H_e$  are subsequently computed, according to the above formulae, which also determine  $A$ ,  $B$ ,  $C$  and  $D$ . In the plots, we shall change the positrons concentration, which leads to recalculate all the physical parameters again. Obviously, by varying the positron concentration, we simultaneously modify all the parameter values used in the plots below.

##### A. Solitary and Explosive/Blowup Excitations

It may be appropriate to point out that the analytical solutions in Sec. III have been obtained for different arbitrary constants  $k, c_0, \dots, c_4$ . One of them is the localized solution

(22), which is a bell-shaped solitary wave solution. Recall that the arbitrary constant  $k$  can be either  $+1$  or  $-1$ . For  $k = -1$ , a positive solitary pulse can propagate and for  $k = +1$ , a negative solitary pulse exist. Note that we have executed the negative solitary pulse since it is not physically correct in the the present model. Figure 2 depicts the QIA solitary pulse for different values of positron concentration  $\mu_p$ , which now determines  $T_{Fe,p}$  (and the ratio  $\sigma$ ) through the charge-neutrality condition. It is found that the amplitude of the soliton pulse decreases by increasing  $\mu_p$ , resulting an increase (decrease) of the electron Fermi temperature  $T_{Fe}$  (quantum diffraction effect  $H_e$ ). Physically, the increase of  $T_{Fe}$  leads to an increase of the electron Fermi energy (viz.  $K_B T_{Fe} = E_{Fe} \equiv (\hbar^2/2m)(3\pi^2 n_{e0})^{2/3}$ ), and as a result the ion Fermi energy should decrease to conserve the energy law. The decrease of the ion Fermi energy decreasing the nonlinearity of the system and hence the height of the soliton pulse shrinks.

It may be interesting to note that for certain values of plasma parameters the solitary pulse convert to an explosive/blowup excitation as shown in Fig. 3. The blowup excitation indicates that an instability in the system can produce due to the effect of the nonlinearity (which in our case depends on the positron concentration  $\mu_p$  and the quantum statistical effects  $\sigma$ ). On the other hand, the magnitude of some quantities (e.g. temperature, pressure, density, etc.) leads to prejudice the balance between the dispersion and the nonlinearity [31]. Therefore, the amplitude may increase to very high values, which gives rise to increasing the electric potential and then accelerate the moving particles.

It is important to notice that Eq. (23) is an explosive/blowup solution, i.e. the potential  $\varphi$  infinitely grows at a finite point (for any fixed  $X, Y, Z \rightarrow X_0, Y_0, Z_0$ ), there exist an  $\xi_0$  at which the solution (23) blowup and thereby we regard the latter as an explosive solution as depicted in Fig. 4.

## B. Shock/Double Layer Excitation

For the shock/double layer solution [32], the boundary condition  $\varphi(\xi) \rightarrow 0$  at  $\xi \rightarrow \infty$  must satisfy. Applying the last boundary condition into Eq. (21), we obtain the double layer solution as

$$\varphi = \varphi_m [1 + \tanh(W_D \xi)], \quad (24)$$

where the amplitude of the double layers is  $\varphi_m = -A_0/2B_0$ , the width is  $W_D = \sqrt{-24\gamma B_0/A_0^2}$ . Here  $v(= -A_0^2/6B_0)$  is the shock wave speed. Notice that  $B_0 < 0$  has to be fulfilled, in order for making the width  $W_D$  real. The numerical analysis in Fig. 5, however, shows that for small positron concentration  $\mu_p$  the dominant situation corresponds to  $B_0 < 0$ , so the double layers may exist. For large positron concentration  $\mu_p$ , double layers cannot occur, since  $B_0 > 0$ . Typically, we have used the plasma density value for white dwarf [11] via  $n_{i0} = 2 \times 10^{32} \text{ cm}^{-3}$  and assume that  $L_z = 0.2$ , which leads to the fact that for negative  $B_0$  (i.e., formation of double layers) the positron concentration  $n_{p0}$  must less than  $1.43308 \times 10^{31} \text{ cm}^{-3}$ . Also, it noted that the narrow range of  $\mu_p$  [corresponding to  $B_0 < 0$ ] will not change the ion gyrofrequency  $\Omega$ . Generally speaking, one can also note from Eq. (24) that the nature of the double layer depends on the sign of  $A_0$ , i.e. for  $A_0 > 0$  a positive double layer exists (viz  $\varphi_m > 0$ ), whereas for  $A_0 < 0$  we would have a negative double layer ( $\varphi_m < 0$ ). For white dwarf plasma parameters, it is found that  $A_0$  is usually greater than zero and then only positive double layers can exist.

Equation (24) describes the double layer potential, which has a well-know profile (cf. Fig. 6). This profile may change due to vary of physical parameters. The dependence of double layer characteristics on the positron concentration  $\mu_p$  [which determines  $T_{Fe,p}$ ,  $H_e$  and  $\sigma$  through the charge-neutrality condition] is depicted in Fig. 7. It is obvious that an increase in the positron concentration  $\mu_p$  shrinks the double layers width but the amplitude increases by increasing  $\mu_p$ .

**It important to note here that in Ref. [33], the soliton excitation in  $e$ - $p$ - $i$  magnetoplasma was investigated but the present work investigates soliton, shock and explosive excitations in  $e$ - $p$ - $i$  magnetoplasma. Therefore, the present model studies another two nonlinear structures, which did not discuss in Ref. [33]. Also, in Ref. [27], the authors used the extended Conte's truncation method to obtain the solitary, explosive, and periodic solutions of the QZK equation. Note that this method gives solitary and explosive excitations described by equation (25) and periodic excitation described by equation (26). Thus, the extended Conte's truncation method cannot predict the shock formation, which may arise due to the presence of weakly double layers. In the present work, we have used generalized expansion method. The later succeeded to describe soliton, explosive, as well as shock excitations. Therefore, the present method**

can be considered as a powerful tool to deal with more general nonlinear partial differential equations.

## V. SUMMARY

To summarize, we have presented the properties of the nonlinear structures QIA waves in a very dense Fermi plasma, composed of the electrons, positrons and positive ions. By employing the reductive perturbation method, an extended quantum ZK equation is derived. The latter has been solved using the generalized expansion method to obtain a set of analytical solutions, which reflects the possibility of propagation of various nonlinear structures (viz. explosive, soliton and shock pulses). We have numerically examined the effects of the positron concentration (which changes the quantum statistics and quantum diffraction parameters through the charge-neutrality condition) on the electrostatic potential excitations, by varying relevant physical parameters. It is found that the amplitudes and widths of the nonlinear structures are significantly affected by the positron concentration, quantum statistical, and quantum tunneling effects. Also, for certain plasma parameters the solitary pulse transforms to blowup pulse. Finally, we stress that this investigation should be useful for understanding the features of the nonlinear structures QIA waves in an electron-positron-ion plasma, such as those in the superdense white dwarfs and in the intense laser-solid matter interaction experiments.

Appendix: The necessary condition to neglect the annihilation process

To neglect the annihilation process, the following inequality must satisfy

$$\frac{1}{\omega_{pe}} \ll T_{ann}, \quad (\text{A1})$$

where  $(1/\omega_{pe})$  is the electron plasma period and  $T_{ann}$  is the annihilation time. For nonrelativistic plasma, the time of annihilation reads [34]

$$T_{ann} = \frac{4}{3\sigma_T n_e c} \left[ \frac{\Theta}{1 + 6\Theta} \right], \quad (\text{A2})$$

where  $\sigma_T$  ( $= 6.65 \times 10^{-25} \text{ cm}^2$ ) is the cross section and  $\Theta$  ( $= K_B T/mc^2$ ) is the temperature range, which satisfy the inequality [34]

$$\alpha^2 < \Theta < 1, \quad (\text{A3})$$

where  $\alpha$  ( $= 7.2974 \times 10^{-3}$ ) is the Fine-structure constant. Equation (A3) can be rewritten in terms of temperature as

$$3 \times 10^5 < T \text{ (K)} < 5.9 \times 10^9 \quad (\text{A4})$$

Inserting Eq. (A2) into (A1), we obtain

$$\Theta > 2.66 \times 10^{-19} n^{1/2}. \quad (\text{A5})$$

Using Eq. (A3) and (A5), one can calculate the range of the density where the annihilation can be ignored

$$3.9 \times 10^{28} < n_e \text{ (cm}^{-3}\text{)} < 1.4 \times 10^{37}. \quad (\text{A6})$$

The quantum effects become important for certain values of density ( $n_{e,p}$ ) and temperature ( $T_{e,p}$ ). The quantum condition  $n_{e,p} \lambda_B^3 \geq 1$  specifies the temperature-density relation, where the quantum effects become important as

$$T_{e,p} \leq 3.2 \times 10^{-11} n_{e,p}^{2/3}. \quad (\text{A7})$$

Using Eq. (A6) with (A7), one can calculate the range of temperature in quantum plasma as

$$3.6 \times 10^8 < T_{e,p} \text{ (K)} < 1.8 \times 10^{14}. \quad (\text{A8})$$

It is clear that the range for neglecting annihilation is well satisfied for white dwarf [see Ref. [11]]. Therefore, the present model can be applicable to the dense white dwarf.

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(a) Also at: Theoretical Physics Group, Department of Physics, Faculty of Science, Mansoura University, Damietta Branch, New Damietta 34517, Egypt. Electronic mail: [refaatsabry@mans.edu.eg](mailto:refaatsabry@mans.edu.eg) and [sabryphys@yahoo.com](mailto:sabryphys@yahoo.com)

(b) Present address: Department of Physics, Faculty of Science-Port Said, Suez Canal University, Egypt. Electronic mail: [wmmoslem@hotmail.com](mailto:wmmoslem@hotmail.com) and [wmm@tp4.rub.de](mailto:wmm@tp4.rub.de)

(c) Also at: Universidade do Vale do Rio dos Sinos-UNISINOS, Av. Unisinos, 950, 93022-000 Sao Leopoldo RS, Brazil. Electronic mail: [ferhaas@tp4.rub.de](mailto:ferhaas@tp4.rub.de)

(d) Electronic mail: [shahid\\_gc@yahoo.com](mailto:shahid_gc@yahoo.com)

(e) Also at: Department of Physics, Umeå University, SE-90187 Umeå, Sweden. Electronic mail: [ps@tp4.rub.de](mailto:ps@tp4.rub.de)

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# Figure Captions

## Figure 1 (color online):

Three-dimensional profile of the solitary pulse [given by Eq. (22)]. A positive solitary pulse for  $k = -1$ ,  $\mu_p = 0.8304$ ,  $\sigma = 1.693$ ,  $\Omega = 0.05$ ,  $H_e = 0.03$ ,  $T = 0$ ,  $Y = 0.1$ ,  $L_x = 0.01$ , and  $L_z = 0.1$ .

## Figure 2 (color online):

Two-dimensional profile of the solitary pulse [given by Eq. (22)]. A positive solitary pulse for  $k = -1$ . For curve A,  $\mu_p = 0.5$ ,  $\sigma = 2.08$ ,  $\Omega = 0.01$ , and  $H_e = 0.0075$ , for curve B,  $\mu_p = 0.75$ ,  $\sigma = 1.75$ ,  $\Omega = 0.0102$ , and  $H_e = 0.0068$  and for curve C,  $\mu_p = 1$ ,  $\sigma = 1.587$ ,  $\Omega = 0.010291$ , and  $H_e = 0.00624$ . Also, we have used  $T = 0$ ,  $X = Y = 0.1$ ,  $L_x = 0.01$ , and  $L_z = 0.1$ .

## Figure 3 (color online):

Three-dimensional profile of the explosive/blowup pulse [given by Eq. (22)]. A positive explosive pulse for  $k = -1$ ,  $\mu_p = 0.6$ ,  $\sigma = 1.9$ ,  $\Omega = 0.0102$ ,  $H_e = 0.0072$ ,  $T = 0$ ,  $Y = 0.1$ ,  $L_x = 0.01$ , and  $L_z = 0.1$ .

## Figure 4:

Three-dimensional profile of the explosive/blowup pulse [given by Eq. (23)], for  $\mu_p = 0.0525$ ,  $\sigma = 7.37$ ,  $\Omega = 0.0257$ ,  $H_e = 0.0081$ ,  $T = 0$ ,  $Y = 0.1$ ,  $L_x = 0.01$ , and  $L_z = 0.2$ .

## Figure 5:

The nonlinear coefficient  $B_0$  is depicted against the positron density  $n_{p0}$  for  $n_{i0} = 2 \times 10^{32}$   $\text{cm}^{-3}$  and  $L_z = 0.2$ . Recall that for  $n_{p0} < 1.43308 \times 10^{31}$   $\text{cm}^{-3}$  the nonlinear coefficient  $B_0 < 0$  and then a shock pulse can propagate.

## Figure 6:

Three-dimensional profile of the shock pulse [given by Eq. (24)], for  $\mu_p = 0.05$ ,  $\sigma = 7.6$ ,  $\Omega = 0.03$ ,  $H_e = 0.008$ ,  $T = 0$ ,  $Y = 0.1$ ,  $L_x = 0.01$ , and  $L_z = 0.2$ .

## Figure 7:

Two-dimensional profile of the shock pulse [given by Eq. (24)]. For curve A,  $\mu_p = 0.05$ ,  $\sigma = 7.6$ ,  $\Omega = 0.03$ ,  $H_e = 0.008$  and for curve B,  $\mu_p = 0.052$ ,  $\sigma = 7.37$ ,  $\Omega = 0.03$ , and  $H_e = 0.0081$ . Here,  $T = 0$ ,  $X = Y = 0.1$ ,  $L_x = 0.1$ , and  $L_z = 0.2$ . Recall that the narrow range of  $\mu_p$  will not affect on the ion-gyrofrequency  $\Omega$ .