Effects of the electron spin on the nonlinear generation of quasi-static magnetic fields in a plasma

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Abstract

Through an extended kinetic model, we study the nonlinear generation of quasi-static magnetic fields by high-frequency fields in a plasma, taking into account the effects of the electron spin. It is found that although the largest part of the nonlinear current in a moderate density, moderate temperature plasma is due to the classical terms, the spin may still give a significant contribution to the magnetic field generation mechanism. Applications of our results are discussed.

I. INTRODUCTION

The low-frequency fields nonlinearly generated by high-frequency waves in a plasma has been the subject of much studies during several decades [1–15]. The focus has been on the ponderomotive force, see e.g. [1, 2], on heating nonlinearities due to collisional effects [3], or on the possibility to generate quasi-static magnetic fields [4–15]. Generation of such fields has proven to be rather sensitive to small deviations from the classical collisionfree Vlasov model. Thus Ref. [11] found that a small collision frequency (described by the Lorenz collision model) was sufficient to significantly alter the predictions from the Vlasov equation. Furthermore, Refs [12, 15] found that weak relativistic effects (with the thermal energy much smaller than the electron rest mass energy), also could alter the nonlinear generation of quasi-static magnetic fields.

In the present paper we will investigate to what extent quantum mechanical effects [16–23, 26], and in particular electron spin effects [23–30], can change the nonlinear low-frequency response to high-frequency fields. A semi-classical model for spin effects generalizing the Vlasov equation was presented decades ago [28]. Recently it was improved to include the magnetic dipole force and the magnetization current due to the spin [29], and finally a fully quantum mechanical model was given [30], reducing to the Wigner equation [17] without spin effects, which can be further reduced to the classical Vlasov equation. Here we start from the long wavelength limit (spatial scales much longer than the thermal de Broglie wavelength) of the full theory [30], where all quantum mechanical effects are directly associated with the spin. A formal expression for the nonlinear low-frequency current is defined for a general geometry. This expression is then evaluated for two special cases with specified wave-vectors and polarization of the high-frequency fields. It is then found that although the classical terms gives the largest contribution to the nonlinear current for a plasma of moderate density and temperature, the spin terms can still contribute significantly to the generation of quasi-static magnetic fields.

The organization of the paper is as follows. In Section II the calculation procedure starting from the classical Vlasov equation is outlined, and the nonlinear low-frequency current corresponding to this case is presented. In Section III the spin kinetic equation is introduced, and our main results starting from this model is derived. In Section IV we investigate the generation of quasi-static magnetic fields by the nonlinear current densities that we have derived. In section V, our results summarized and discussed. Finally, the Appendix addresses the possible significance of quantum mechanical effects left out in the previous calculations.

II. CLASSICAL KINETIC MODEL

In order to outline our method of calculating the nonlinear current, we shall first consider the nonlinear mixing of two high-frequency waves $(\omega_1, \mathbf{k}_1)$ and $(\omega_2, \mathbf{k}_2)$ in a collisionless plasma. Considering only electron motion we describe the evolution of the perturbation f of the electron distribution function by means of the classical Vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla f + \frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial F_0}{\partial \mathbf{v}} = -\frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f}{\partial \mathbf{v}} \tag{1}$$

where q/m is the electron charge to mass ratio and $F_0(\mathbf{v})$ is the unperturbed velocity distribution function normalized such that $n_0 = \int F_0 d^3v$. For simplicity we assume that F_0 is Maxwellian, i.e. $F_0 \sim \exp(-v^2/2v_t^2)$. Relativistic effects are neglected. For notational convenience we assume that ω_2 is negative. The nonlinear mixing between the two high-frequency waves thus yields a low-frequency response at $(\omega, \mathbf{k})$ where $\omega = \omega_1 + \omega_2$ and $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$. Replacing the right hand side of equation (1) by $(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_1)\cdot(\partial f_2/\partial\mathbf{v}) + (1 \leftrightarrow 2)$ where $(1 \leftrightarrow 2)$ means interchange of indices 1 and 2, and $f_2 \approx -(iq/m)\mathbf{E}_2\cdot(\partial F_0/\partial\mathbf{v})/(\omega_2 - \mathbf{k}_2\cdot\mathbf{v})$, we then obtain the nonlinear and classical part of the generated low-frequency current density $\mathbf{J}_{\rm cl}$ as

$$\mathbf{J}_{\rm cl} = -\frac{iq^2}{m}\int \frac{\mathbf{v}}{(\omega - \mathbf{k}\cdot\mathbf{v})}\left[(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_1)\cdot\frac{\partial f_2}{\partial \mathbf{v}} + (1 \leftrightarrow 2)\right] d\mathbf{v} \tag{2}$$

which can be directly rewritten in the form

$$\begin{aligned}\mathbf{J}_{\rm cl} = {} & \frac{q^3}{m^2 v_t^2}\int \frac{\mathbf{v}F_0}{\omega - \mathbf{k}\cdot\mathbf{v}}\left[\frac{(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_1)\cdot\mathbf{E}_2}{(\omega_2 - \mathbf{k}_2\cdot\mathbf{v})} - \right. \\ & \left. \frac{\mathbf{E}_1\cdot\mathbf{v}\mathbf{E}_2\cdot\mathbf{v}}{v_t^2(\omega_2 - \mathbf{k}_2\cdot\mathbf{v})} + \frac{\mathbf{k}_2\cdot(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_1)\mathbf{E}_2\cdot\mathbf{v}}{(\omega_2 - \mathbf{k}_2\cdot\mathbf{v})^2} + (1 \leftrightarrow 2)\right] d^3v\end{aligned} \tag{3}$$

Using the fact that the phase velocities of the high-frequency waves are much larger than the thermal velocity v_t we can adopt $\mathbf{k}_{1,2}\cdot\mathbf{v}/\omega_{1,2}$ and $\omega/\omega_{1,2}$ as expansion parameters and obtain, up to order $(\omega/\omega_{1,2})^3$

$$\begin{aligned}\mathbf{J}_{\rm cl} = {} & \frac{q^3}{m^2 v_t^2 \omega_1 \omega_2}\int \mathbf{v}F_0\left\{\frac{\omega - 2\mathbf{k}\cdot\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}\left[\mathbf{E}_1\cdot\mathbf{E}_2\left(1 + \frac{\mathbf{k}_1\cdot\mathbf{v}}{\omega_1} + \frac{\mathbf{k}_2\cdot\mathbf{v}}{\omega_2}\right) + \right.\right. \\ & \left.\left. \mathbf{v}\cdot\mathbf{E}_1\frac{\mathbf{k}_1\cdot\mathbf{E}_2}{\omega_1} + \mathbf{v}\cdot\mathbf{E}_2\frac{\mathbf{k}_2\cdot\mathbf{E}_1}{\omega_2}\right] - \frac{\mathbf{v}\cdot\mathbf{E}_1\mathbf{v}\cdot\mathbf{E}_2}{v_t^2}\left(\frac{\mathbf{k}_1\cdot\mathbf{v}}{\omega_1} + \frac{\mathbf{k}_2\cdot\mathbf{v}}{\omega_2}\right)\right\} d^3v\end{aligned} \tag{4}$$

The first term of Eq. (4) (proportional to $\mathbf{E}_1 \cdot \mathbf{E}_2$) is one order larger in the expansion made, but other terms are important as they are the ones contributing the generation of quasi-static magnetic fields.

III. SPIN KINETIC MODEL

Next we consider an extended distribution function $f(\mathbf{r}, \mathbf{v}, \mathbf{s}, t)$, where $\mathbf{s}$ is a spin vector with fixed length, describing the orientation of the electron spin. On semi-classical grounds it can be shown [29] that $df/dt = 0$, where d/dt is a total time derivative following the generalized particle orbit, i.e. including the spin evolution. Using the Heisenberg equation of motion to get $d\mathbf{s}/dt$, a semi-classical model including the spin degree of freedom is found [29]. With certain corrections, such a model can be derived as the long scale limit (scale lengths much longer than the thermal de Broglie wavelength) of a fully quantum mechanical treatment [30]. The evolution equation in this regime is then found to be

$$\partial_t f + \mathbf{v}\cdot\nabla f + \left[\frac{q}{m}\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) + \frac{2\mu_e}{m}\nabla\left(\mathbf{s}\cdot\mathbf{B}\right) + \frac{3\mu_e}{2m}\nabla\left(\mathbf{B}\cdot\nabla_{\mathbf{s}}\right)\right]\cdot\nabla_{\mathbf{v}} f + \frac{2\mu_e}{\hbar}\left(\mathbf{s}\times\mathbf{B}\right)\cdot\nabla_s f = 0 \tag{5}$$

where we here use the convention that $|\mathbf{s}| = 1$. The difference from the semi-classical theory deduced from $df/dt = 0$ is twofold. Firstly, the constant coefficients of the spin terms differ somewhat (this is compensated by different coefficients in the spin current given below, such that also the semi-classical model is energy conserving). Secondly, the term proportional to $\mathbf{B}\cdot\nabla_{\mathbf{s}}$ has no semi-classical counterpart. This term can be viewed as part of the magnetic dipole force, which account for the fact that a quantum mechanical probability distribution of a single particle spin is always smeared out, as opposed to a classical spin (or magnetic dipole moment) which has a unique direction. The spin kinetic model is completed by Maxwell's equations with the current density

$$\mathbf{J} = \mathbf{J}_f + \mathbf{J}_m = q\int \mathbf{v} f d^3v\, d\Omega_s + \mu_e \int \mathbf{s} f d^3v\, d\Omega_s \tag{6}$$

where $\mathbf{J}_f$ and $\mathbf{J}_m$ is the free part and magnetization (spin) part of the current density, respectively. Using spherical coordinates for the spin angles we have $d\Omega_s = \sin\theta_s d\theta_s d\varphi_s$.

Next we follow the steps of section II, and write the nonlinear high-frequency source terms (due to $(\omega_1, \mathbf{k}_1)$ and $(\omega_2, \mathbf{k}_2)$) on the right hand side. Since the zero order distribution function F_0 is isotropic and independent of $\mathbf{s}$ we find

$$\partial_t f + \mathbf{v}\cdot\nabla f + \left[\frac{q}{m}\mathbf{E} + \frac{2\mu_e}{m\hbar}\nabla\left(\mathbf{s}\cdot\mathbf{B}\right)\right]\cdot\nabla_{\mathbf{v}}F_0 = -\left[\frac{q}{m}\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right) + \frac{2\mu_e}{m\hbar}\nabla\left(\mathbf{s}\cdot\mathbf{B}\right) - \frac{3\mu_e\hbar}{2m}\nabla\left(\mathbf{B}\cdot\nabla_{\mathbf{s}}\right)\right]\cdot\nabla_{\mathbf{v}}$$
$$-\frac{2\mu_e}{\hbar}\left(\mathbf{s}\times\mathbf{B}\right)\cdot\nabla_s f. \tag{7}$$

Following the same procedure as in the classical case we consider a Maxwellian background distribution and use the notation

$$\omega = \omega_1 + \omega_2, \tag{8}$$

$$\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2. \tag{9}$$

Since the zero order distribution function is isotropic by assumption, the first order distribution function can be approximated as

$$f_{1,2} \approx \frac{-i\left(q\mathbf{E}_{1,2} + \frac{2\mu_e}{\hbar}\nabla(\mathbf{s}\cdot\mathbf{B}_{1,2})\right)\cdot\partial_{\mathbf{v}}F_0}{m(\omega_{1,2} - \mathbf{k}_{1,2}\cdot\mathbf{v})}, \tag{10}$$

and we get the free nonlinear current density

$$\mathbf{J}_{\mathrm{f}} = -iq\int d^3v\, d\Omega_s \frac{\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}\left\{\left[\frac{q}{m}\left(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_1\right) + \frac{2\mu_e}{m\hbar}\nabla\left(\mathbf{s}\cdot\mathbf{B}\right) + \frac{3\mu_e\hbar}{2m}\nabla\left(\mathbf{B}\cdot\nabla_{\mathbf{s}}\right)\right]\cdot\partial_{\mathbf{v}}f_2\right.$$
$$\left. + \frac{2\mu_e}{\hbar}\left(\mathbf{s}\times\mathbf{B}_1\right)\cdot\partial_{\mathbf{s}}f_2 + (1\leftrightarrow 2)\right\}, \tag{11}$$

and the nonlinear magnetization current

$$\mathbf{J}_{\mathrm{m}} = -i\nabla\times\int d^3v\, d\Omega_s \frac{\mathbf{s}}{\omega - \mathbf{k}\cdot\mathbf{v}}\left\{\left[\frac{q}{m}\left(\mathbf{E}_1 + \mathbf{v}\times\mathbf{B}_1\right) + \frac{2\mu_e}{m\hbar}\nabla\left(\mathbf{s}\cdot\mathbf{B}\right) + \frac{3\mu_e\hbar}{2m}\nabla\left(\mathbf{B}\cdot\nabla_{\mathbf{s}}\right)\right]\cdot\partial_{\mathbf{v}}f_2\right.$$
$$\left. + \frac{2\mu_e}{\hbar}\left(\mathbf{s}\times\mathbf{B}_1\right)\cdot\partial_{\mathbf{s}}f_2 + (1\leftrightarrow 2)\right\}. \tag{12}$$

A general geometry with arbitrary polarizations and wavevectors of the high-frequency waves leads to extremely lengthy algebra, and is not within the scope of the present paper. Thus as our first special case we consider parallel polarization and propagation. For definiteness we choose the geometry

$$\mathbf{k}_{1,2} = k_{1,2}\hat{\mathbf{z}}, \tag{13}$$

$$\mathbf{B}_{1,2} = B_{1,2}\hat{\mathbf{y}}, \tag{14}$$

$$\mathbf{E}_{1,2} = E_{1,2}\hat{\mathbf{x}}. \tag{15}$$

The high-frequency waves may be co-propagating or counter-propagating depending on the sign of $k_{1,2}$. In this case subtracting the classical contribution $\mathbf{J}_{\rm cl}$ we obtain that the free part and the magnetization part of the nonlinear current due to the spin, denoted $\mathbf{J}_{\rm sp}$ combines to

$$\begin{aligned}\mathbf{J}_{\rm sp} =& -q\frac{3\mu_e^2}{m^2 v_t^2}k_1 k_2 B_1 B_2 \int d^3v\, d\Omega_s\, \frac{\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}F_0 \\ &\times \left[\frac{1}{(\omega_2 - k_2 v_z)}\left(1 - \frac{v_z^2}{v_t^2} + \frac{k_2 v_z}{\omega_2 - k_2 v_z}\right) + \frac{1}{(\omega_1 - k_1 v_z)}\left(1 - \frac{v_z^2}{v_t^2} + \frac{k_1 v_z}{\omega_1 - k_1 v_z}\right)\right] \end{aligned} \quad (16)$$

which for $v_t \ll \omega_1/k_1, \omega_2/k_2$ reduces to

$$\mathbf{J}_{\rm sp} = -q\frac{3\mu_e^2}{m^2 v_t^2}k_1 k_2 B_1 B_2 \int d^3v\, d\Omega_s\, \frac{\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}F_0 \left[\frac{1}{\omega_2}\left(1 - \frac{v_z^2}{v_t^2} + \frac{k_2 v_z}{\omega_2}\right) + \frac{1}{\omega_1}\left(1 - \frac{v_z^2}{v_t^2} + \frac{k_1 v_z}{\omega_1}\right)\right]. \quad (17)$$

Since $\mathbf{k}$ is along $\hat{\mathbf{z}}$, we note that $\mathbf{J}_{\rm sp}$ in (17) will be in the $\hat{\mathbf{z}}$-direction. This is the same direction as the classical contribution found from (4). Furthermore, for moderate temperatures and densities the classical contribution will be larger than that due to the spin, and hence the result given by (17) is merely a small correction, at least if we limit ourselves to parameters of temperature and density corresponding to laboratory and space plasmas.

Next we modify our special case and consider the polarization of the waves to be perpendicular. As it turns out, the spin contribution to the nonlinear current then vanishes, although there will be a significant magnetization in the $\hat{\mathbf{z}}$-direction. If we also modify the wavevectors slightly, this nonzero magnetization contributes to a nonlinear current density. This means that we consider the following geometry:

$$\mathbf{E}_1 = E_1\hat{\mathbf{x}}, \quad (18)$$

$$\mathbf{E}_2 = -E_2\hat{\mathbf{y}}, \quad (19)$$

$$\mathbf{k}_1 = k_1\hat{\mathbf{z}}, \quad (20)$$

$$\mathbf{k}_2 = k_2\hat{\mathbf{z}} + \Delta k_2\hat{\mathbf{x}}, \quad (21)$$

$$\mathbf{B}_1 = B_1\hat{\mathbf{y}}, \quad (22)$$

$$\mathbf{B}_2 = B_2\hat{\mathbf{x}} + \Delta B_2\hat{\mathbf{z}}. \quad (23)$$

To limit the algebra, the deviation from parallel propagation is considered to be small, and we will thus only consider terms to first order in Δk_2 or ΔB_2. By Faradays law we note that $\Delta k_2/k_2 = \Delta B_2/B_2$. When we calculate the nonlinear currents in this geometry it turns out that the free part of the current vanishes, i.e. $\mathbf{J}_{\rm sp} = \mathbf{J}_{\rm sp-m}$ and the nonlinear spin current density reduces to

$$\mathbf{J}_{\rm sp} = -\hat{\mathbf{y}}\frac{16\mu_e^3}{mv_t^2\hbar}\pi\,\Delta k_2\int d^3v\,B_1B_2\frac{F_0v_z}{\omega-\mathbf{k}\cdot\mathbf{v}}\left[\frac{k_1\left(\frac{k}{k_2}+1\right)}{\omega_1-\mathbf{k}_1\cdot\mathbf{v}}+\frac{k_2\left(\frac{k}{k_2}-1\right)}{\omega_2-\mathbf{k}_2\cdot\mathbf{v}}\right] \tag{24}$$

which vanish if we let $\Delta k_2 \to 0$. Taking the limit of low temperature, i.e $v_t \ll \omega/k$, this further simplifies to

$$\mathbf{J}_{\rm sp} = -\hat{\mathbf{y}}\frac{16\mu_e^3}{m\hbar}\pi\,\Delta k_2 n_0\frac{k}{\omega^2}\left[\frac{k_1}{\omega_1}\left(\frac{k}{k_2}+1\right)+\frac{k_2}{\omega_2}\left(\frac{k}{k_2}-1\right)\right]B_1B_2 \tag{25}$$

The geometry in this special case is of more interest, as we will find that the spin-contribution can give a larger contribution to the generation of quasi-static magnetic fields than the free current, also for relatively modest plasma temperatures and densities. This issue will be investigated further in section IV.

A. A general isotropic background distribution

The above results can easily be generalized to be valid for any background distribution that is a function of v^2, i.e. isotropic, which is of interest e.g. for a dense plasma when the thermodynamic equilibrium distribution is a Fermi-Dirac distribution rather than a Maxwellian. With this more general background distribution we get in the case of parallel polarization the nonlinear spin current

$$\begin{aligned}\mathbf{J}_{\rm sp} =&q\frac{6\mu_e^2}{m^2}k_1k_2B_1B_2\int d^3v\,d\Omega_s\,\frac{\mathbf{v}}{\omega-\mathbf{k}\cdot\mathbf{v}}\\ &\times\left[\frac{1}{(\omega_2-k_2v_z)}\left(1-\frac{v_z^2}{v_t^2}+\frac{k_2v_z}{\omega_2-k_2v_z}\right)+\frac{1}{(\omega_1-k_1v_z)}\left(1-\frac{v_z^2}{v_t^2}+\frac{k_1v_z}{\omega_1-k_1v_z}\right)\right]\frac{\partial}{\partial(v^2)}F_0,\end{aligned} \tag{26}$$

reducing to

$$\mathbf{J}_{\rm sp} = q\frac{6\mu_e^2}{m^2}k_1k_2B_1B_2\int d^3v\,d\Omega_s\,\frac{\mathbf{v}}{\omega-\mathbf{k}\cdot\mathbf{v}}\left[\frac{1}{\omega_2}\left(1-\frac{v_z^2}{v_t^2}+\frac{k_2v_z}{\omega_2}\right)+\frac{1}{\omega_1}\left(1-\frac{v_z^2}{v_t^2}+\frac{k_1v_z}{\omega_1}\right)\right]\frac{\partial}{\partial(v^2)}F_0 \tag{27}$$

for the limit $v_t \ll \omega_1/k_1, \omega_2/k_2$.

In the case of orthogonal polarization in the same way we obtain

$$\mathbf{J}_{\rm sp} = \hat{\mathbf{y}}\frac{32\mu_e^3}{m\hbar}\pi\,\Delta k_2\int d^3v\,B_1B_2\frac{v_z}{\omega-\mathbf{k}\cdot\mathbf{v}}\left[\frac{k_1\left(\frac{k}{k_2}+1\right)}{\omega_1-\mathbf{k}_1\cdot\mathbf{v}}+\frac{k_2\left(\frac{k}{k_2}-1\right)}{\omega_2-\mathbf{k}_2\cdot\mathbf{v}}\right]\frac{\partial}{\partial(v^2)}F_0 \tag{28}$$

and in the low temperature limit this reduces to (25).

IV. MAGNETIC FIELD GENERATION

To demonstrate the significance of the spin contribution in the nonlinear current density we write Ampere's law as

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mu_0\left(\mathbf{J}_{\rm l}(\omega,\mathbf{k})+\mathbf{J}_{\rm nl}(\omega,\mathbf{k})\right)-\frac{i\omega}{c^2}\mathbf{E}(\omega,\mathbf{k}). \tag{29}$$

Next we write the linear current density as $\mathbf{J}_{\rm l}(\omega,\mathbf{k}) = \sigma(\omega,\mathbf{k})\mathbf{E}(\omega,\mathbf{k})$, where $\sigma(\omega,\mathbf{k})$ is the linear conductivity. Combining (29) with Faraday's law, we immediately find the low frequency magnetic field generated by the nonlinear current as

$$\mathbf{B}(\omega,\mathbf{k}) = \frac{i\mu_0\mathbf{k}\times\mathbf{J}_{\rm nl}}{D_{\rm em}(\omega,\mathbf{k})} = \frac{i\mu_0\mathbf{k}\times(\mathbf{J}_{\rm cl}+\mathbf{J}_{\rm sp})}{D_{\rm em}(\omega,\mathbf{k})} \tag{30}$$

where $D_{\rm em}(\omega,\mathbf{k}) = \omega^2 - k^2c^2 - i\omega\sigma(\omega,\mathbf{k}) \simeq \omega^2 - k^2c^2 - \omega_p^2$. Eq. (30) thus contains both the classical contribution from (4) as well as the spin contribution from (24). To shed further light on this expression we evaluate (30) in the low temperature regime $kv_{th} \ll \omega$, in which case the expression simplifies to

$$\mathbf{B}(\omega,\mathbf{k}) = -\frac{i\mu_0 k}{D_{em}(\omega,\boldsymbol{k})}n_0\frac{B_1B_2}{m}\frac{\Delta k_2}{\omega_2}\left\{\frac{q^3v_t^2(k_1^2+k_2^2)}{mk_1k_2\omega_2}+\frac{16\mu_e^3\pi k_2}{\hbar}\left[\frac{k_1}{\omega_1}\left(\frac{k}{k_2}+1\right)+\frac{k_2}{\omega_2}\left(\frac{k}{k_2}-1\right)\right]\right\} \tag{31}$$

Thus we see that the spin contribution to the magnetic field generation (proportional to $\mu_e^3/\hbar$) dominates in the regime

$$\hbar^2 k_1 k_2 \gtrsim \frac{m^2v_{th}^2}{2} \tag{32}$$

whereas the classical contribution dominates otherwise. A case of experimental interest clearly includes two high-frequency sources. However, we can note that our results also are of relevance for a single source, where $(\omega_1,\mathbf{k}_1)$ and $(\omega_2,\mathbf{k}_2)$ represents different spectral components of a focused pulse. In this case typically $|\mathbf{k}| \ll |\mathbf{k}_{1,2}|$. For the condition (32) it does not matter whether two pulses or a single source is used. Furthermore, for current laser plasma experiments with lasers in the optical regime, it is clear from (32) that the classical contribution will dominate. However, in case laser-plasma experiments with an X-Fel source such as that being built in DESY is made [37], $|\mathbf{k}_{1,2}| \sim 6\times10^9\,\mathrm{m}^{-1}$, and the quantum spin effects can be of importance for magnetic field generation for plasma temperatures $T \lesssim 2\times10^4$K.This condition have been derived for the case of a specified geometry, and it should be noted that the results may differ in case polarizations and/or the directions of wave vectors are changed.

V. SUMMARY AND DISCUSSION

In the present paper we have studied the nonlinear current density generated by high-frequency waves in a plasma, with a focus on the contribution emanating from the electron spin. The largest part of the current density is usually associated with the classical ponderomotive effect. However, it is found that although the largest part of the nonlinear current in a moderate density, moderate temperature plasma is due to the classical terms, the spin may still give a significant contribution to the magnetic field generation mechanism. For the geometry considered here, the condition needed for spin effects to be important require short-wavelength sources, of the order of the x-ray regime. Besides the quantum effects due to spin considered here, there is also particle dispersive quantum effects. Although a thorough consideration of such effects is still to be made, our calculations outlines here indicate that particle dispersive terms may be of comparable importance for the nonlinear current density. Thus there still remains much research in this area to be made.

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Appendix: Particle dispersive effects

The quantum effects due to spin that we have considered here should be compared to the ones due to particle dispersive effects. Such effects can be described by the Wigner function, that reduces to the classical distribution function whenever the thermal de Broglie wavelength is small compared to the scale lengths of the problem. The quantum corrections due to this in the kinetic equation scale as $\hbar^2$, whereas e.g. the magnetic dipole force due to the spin scale as $\hbar$. However, this does not necessarily mean that the lowest order quantum corrections is due to the spin, since for a spin independent zero order distribution function, the spin term at one place (e.g. the magnetic dipole force) always need to be combined with another spin effect (e.g. the magnetization current) to produce a non-vanishing nonlinear current. Thus both the spin effects and particle dispersive effects produce quantum corrections that are proportional to $\hbar^2$ to lowest order. While our main focus of the present paper are the corrections due to the spin, we will here briefly outline how to obtain the quantum correction due to particle dispersive effects. The general evolution equation containing both spin and particle dispersive effects was derived in Ref. [30]. Here we take that evolution equation, drop the spin effects considered above, and keep the particle dispersive effects in a weak quantum expansion (with the thermal de Broglie wavelength over the characteristic scale length as expansion parameter), where only terms up to $\hbar^2$ are kept. The governing equation then reads

$$\begin{aligned}
&\frac{\partial}{\partial t} f + \mathbf{v} \cdot \nabla_x f + \frac{q}{m} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v f \\
=&\frac{\hbar^2}{24m^2} \left\{ \frac{q}{m} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v \left(\overleftarrow{\nabla}_x \cdot \overrightarrow{\nabla}_v \right)^2 - 2 \left[\frac{q}{m} \mathbf{B} \times \nabla_v \left(\overleftarrow{\nabla}_x \cdot \overrightarrow{\nabla}_v \right) \right] \cdot \left(\frac{q}{m} \mathbf{B} \times \nabla_v + \nabla_x \right) \right\} f
\end{aligned} \tag{A.1}$$

Defining

$$f = F_0 + \tilde{f} \tag{A.2}$$

where F_0 is the background distribution and $\tilde{f}$ is the perturbed distribution function, we can separate (7) in linear and nonlinear terms:

$$\begin{aligned}
&\frac{\partial}{\partial t} f + \mathbf{v} \cdot \nabla_x f + \frac{q}{m} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v F_0 - \frac{\hbar^2 q}{24m^3} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v \left(\overleftarrow{\nabla}_x \cdot \overrightarrow{\nabla}_v \right)^2 F_0 \\
&= - \frac{q}{m} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v \tilde{f} + \frac{\hbar^2}{24m^2} \left\{ \frac{q}{m} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v \left(\overleftarrow{\nabla}_x \cdot \overrightarrow{\nabla}_v \right)^2 \tilde{f} \right. \\
&\quad \left. - 2 \left[\frac{q}{m} \mathbf{B} \times \nabla_v \left(\overleftarrow{\nabla}_x \cdot \overrightarrow{\nabla}_v \right) \right] \cdot \left(\frac{q}{m} \mathbf{B} \times \nabla_v F_0 + \nabla_x \tilde{f} \right) \right\}
\end{aligned} \tag{A.3}$$

Assuming two waves as in previous calculations, and a Maxwellian background distribution we have

$$f_{1,2} \approx -\frac{iq(\mathbf{E}_{1,2} + \mathbf{v} \times \mathbf{B}_{1,2}) \cdot \nabla_v \left[1 - \frac{\hbar^2}{24m^2} (i\mathbf{k}_{1,2} \cdot \overrightarrow{\nabla}_v)^2 \right] F_0}{m(\omega_{1,2} - \mathbf{k}_{1,2} \cdot \mathbf{v})}. \tag{A.4}$$

where we separated the perturbed distribution function into a linear part $f_{1,2}$ and a nonlinear part f_{nl}. The rest of the calculations can be performed as in section III, although the algebra gets extremely complicated in general. A thorough treatment of particle dispersive effects is beyond the scope of the present paper, but we will nevertheless point out two conclusions. Firstly, that the particle dispersive effects can be comparable in magnitude to the spin contributions. Secondly, although these two quantum effects may be comparable, they do not typically cancel, as the spin current for parallel propagation parallel polarization scales as $\propto \hbar^2 k_1 k_2 B_1 B_2$ (see eq. (17)), whereas the corresponding scaling can be shown to be

$\propto \hbar^2 B_1 B_2/k_1 k_2$ for particle dispersive effects. Thus we conclude that the contribution of particle dispersive effects to the nonlinear current density is worthy of consideration in its own right, but we should not expect such contributions to cancel those due to the spin.

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[37] See e.g., the technical specification given at the homepage of the European X-ray laser project XFEL (http://xfel.desy.de/).